

Mechanisms and Materials for Odd-Frequency Superconductivity

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Outline



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- Introduction to odd-frequency (ω) superconductivity
 - What?
 - Where and how?
- Weyl nodal loop semimetals
 - Optimal odd- ω Josephson junctions
- Multiband systems
 - Simple two-band/orbital superconductors
 - Odd- ω and Kerr effect in Sr_2RuO_4 (and UPt_3)
 - Dominating odd- ω pairing in doped topological insulator Bi_2Se_3

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Odd-Frequency Superconductivity

- What?
- Where and how?

Superconductivity



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Ordered states → symmetry of order parameter, Δ

Superconductivity: $\Delta \sim F = \langle \psi_\alpha \psi_\beta \rangle$ Expectation value of forming a Cooper pair

Fermi-Dirac
statistics

- spin-singlet, *s*-wave
 $\uparrow\downarrow - \downarrow\uparrow$ 
- spin-triplet, *p*-wave
 $\uparrow\downarrow + \downarrow\uparrow$
 $\uparrow\uparrow, \downarrow\downarrow$ 

Superconductivity



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Ordered states → symmetry of order parameter, Δ

Superconductivity: $\Delta \sim F = \langle \psi_\alpha(t) \psi_\beta(0) \rangle$ ($t \leftrightarrow \omega$, frequency)

Fermi-Dirac
statistics

- even- ω , spin-singlet, s-wave
 $\uparrow\downarrow - \downarrow\uparrow$ 
- odd- ω , spin-triplet, s-wave
 $\uparrow\downarrow + \downarrow\uparrow$
 $\uparrow\uparrow, \downarrow\downarrow$ 



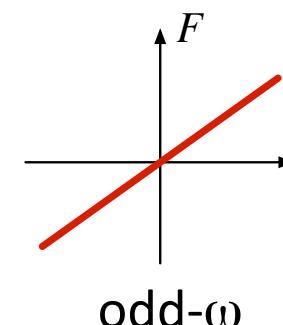
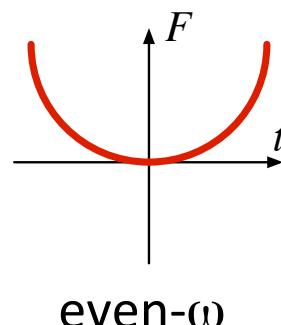
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Odd- ω Pairing

BCS order parameter: $F(\mathbf{r}, t; \mathbf{r}', t' \rightarrow t) = \langle \psi(\mathbf{r}, t) \psi(\mathbf{r}', t' \rightarrow t) \rangle$

vanishes for odd- ω pairing

Equal-time odd- ω order parameter: $\left. \frac{dF(\mathbf{r}, t; \mathbf{r}', t')}{dt} \right|_{t \rightarrow t'}$



Brief History



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Bulk, intrinsic phase:

- 1974, Berezinskii: spin-triplet *s*-wave phase in ${}^3\text{He}$
- 1991, Kirkpatrick&Belitz: Disorder-induced spin-triplet *s*-wave SC
- 1992, Balatsky&Abrahmas: spin-singlet *p*-wave SC
- 1994-5, Coleman,Miranda,Tsvelik: Heavy fermions compounds, Majorana fermions
- Complicated models (e.g. composite condensates) and questions about stability (e.g. paramagnetic Meissner effect)

In this talk:

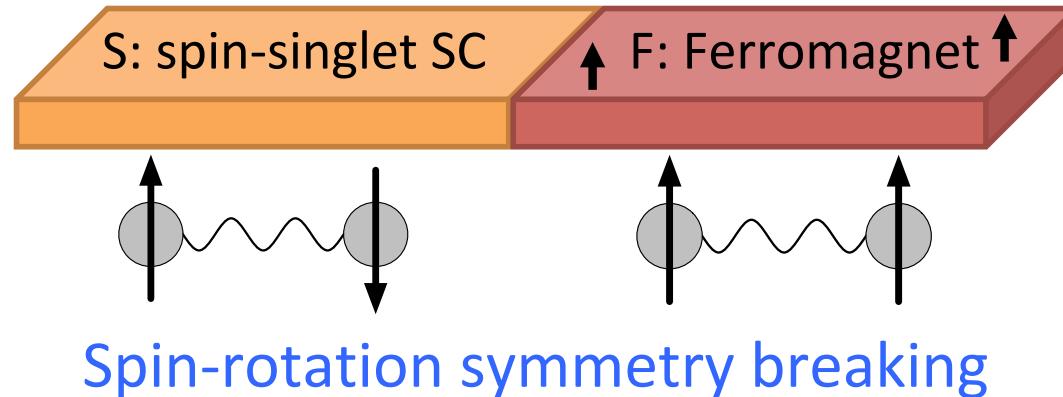
- Heterostructures or multiband systems with conventional order parameter

Reviews: Bergeret, Volkov, Efetov RMP 77, 1321 (2005), Balatsky and Linder, arXiv:1709.03986

SF Interface



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Spin-singlet s -wave SC \rightarrow odd- ω spin-triplet s -wave pairing

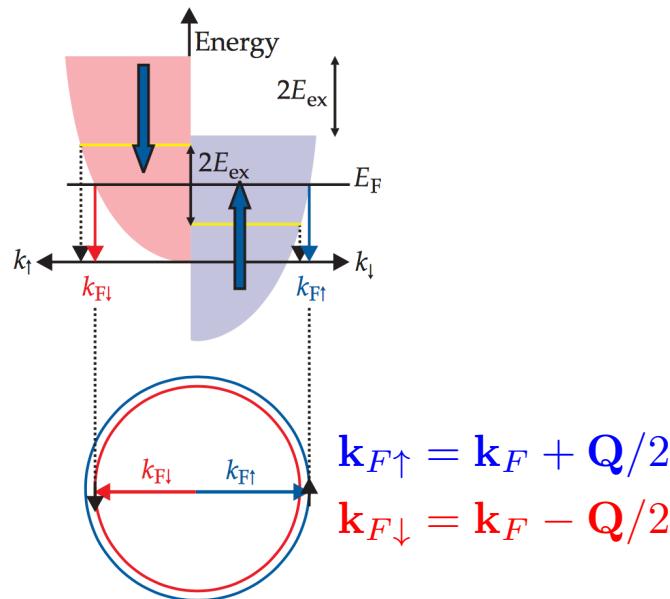
- Measured long-range superconducting proximity effect in F
- s -wave = disorder robust

Bergeret et al, PRL 86, 3140 (2001), Eschrig and Löfwander, Nat. Phys. 4, 138 (2007)



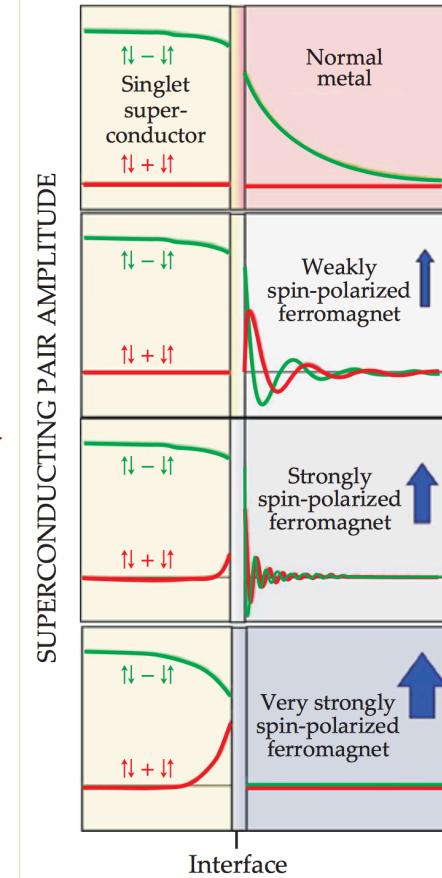
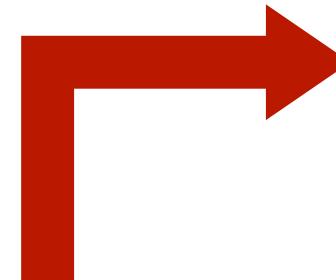
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Superconductivity in FMs



$$(\uparrow\downarrow - \downarrow\uparrow) \rightarrow (\uparrow\downarrow e^{i\mathbf{Q}\cdot\mathbf{R}} - \downarrow\uparrow e^{-i\mathbf{Q}\cdot\mathbf{R}}) = \\ = (\uparrow\downarrow - \downarrow\uparrow) \cos(\mathbf{Q} \cdot \mathbf{R}) + i(\uparrow\downarrow + \downarrow\uparrow) \sin(\mathbf{Q} \cdot \mathbf{R})$$

FFLO state

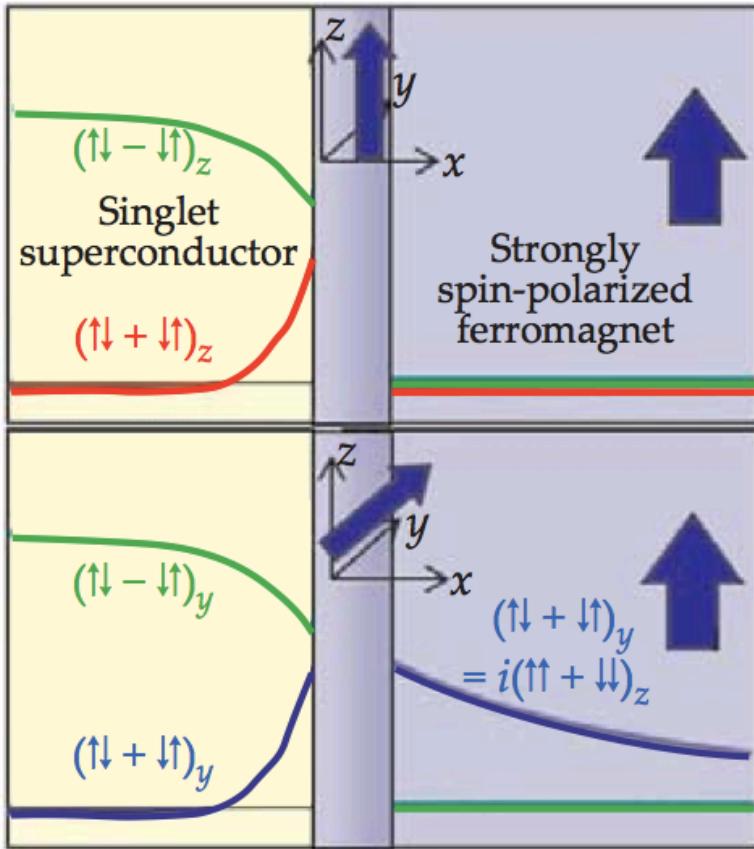


Eschrig, Phys. Today 64, 43 (2011)

Physics at the SF Interface



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Spin-singlet → Mixed-triplet
if only one magnetization direction

Spin-singlet → Mixed-triplet → Equal-triplet
if
Non-collinear magnetization:
Two different magnetic layers
Spin-orbit coupled interface
Helical ferromagnet

Experimental Signatures

- Long-range proximity effect in diffusive F
- Zero-energy peaks in DOS (sometimes)
- Paramagnetic Meissner effect (sometimes)
- Changes in critical temperature in FSF structures

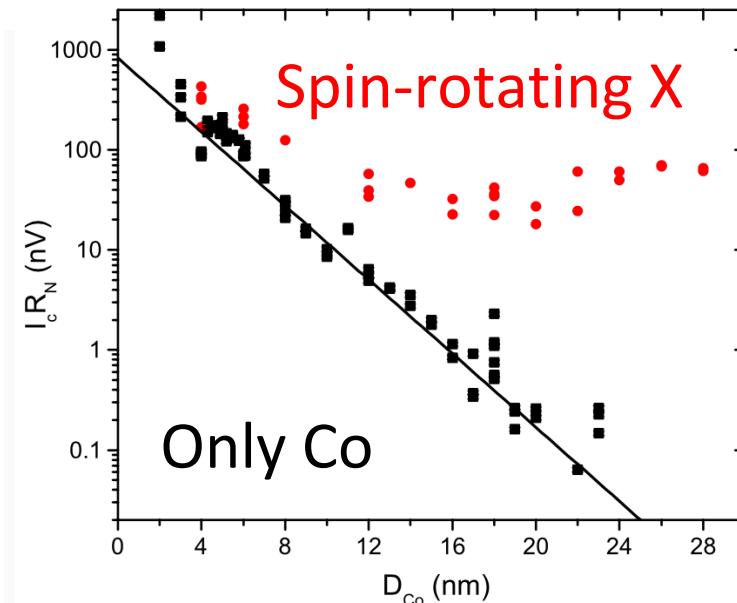
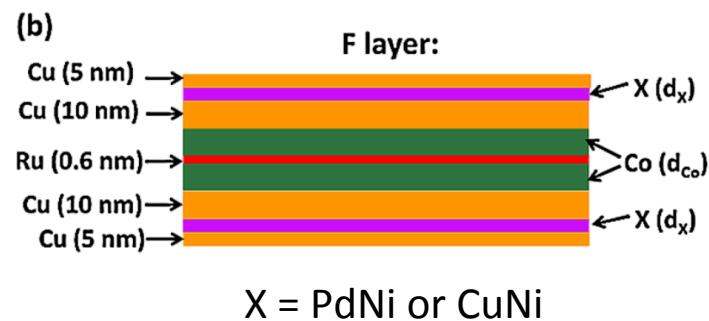
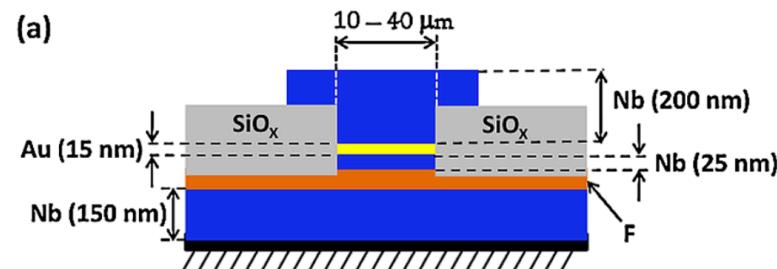
Large body of experiments supporting
odd- ω pairing in SF junctions
(but no direct measurements)



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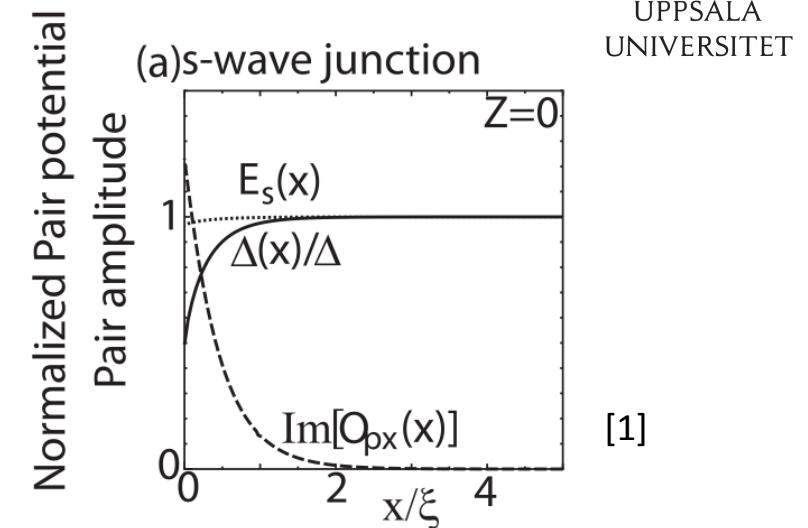
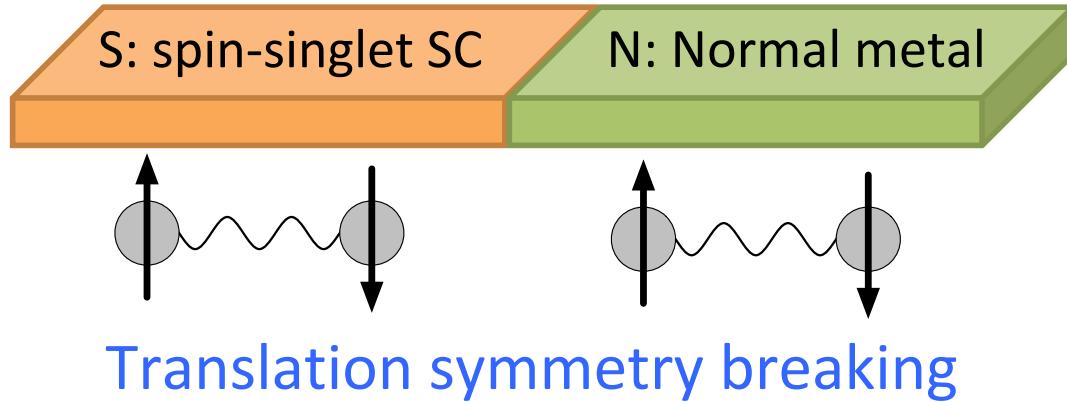
Experimental Signatures

Long-range proximity effect



Long-range proximity effect only
for non-collinear magnetization

SN Interface



Spin-singlet *s*-wave SC \rightarrow odd- ω spin-singlet *p*-wave pairing

- Only high-transparency junctions

[1]: Tanaka et al, PRL 99, 037005 (2007)

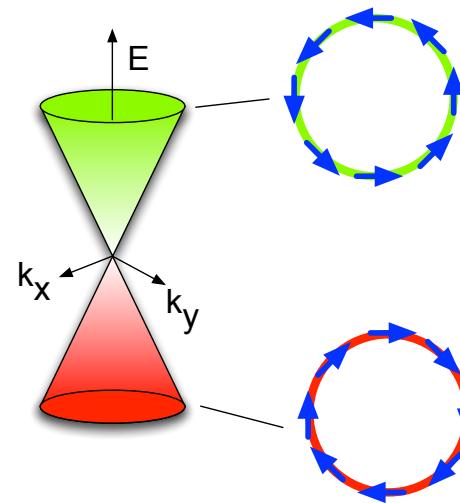


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Topological Insulator Surface

Surface state of a topological insulator

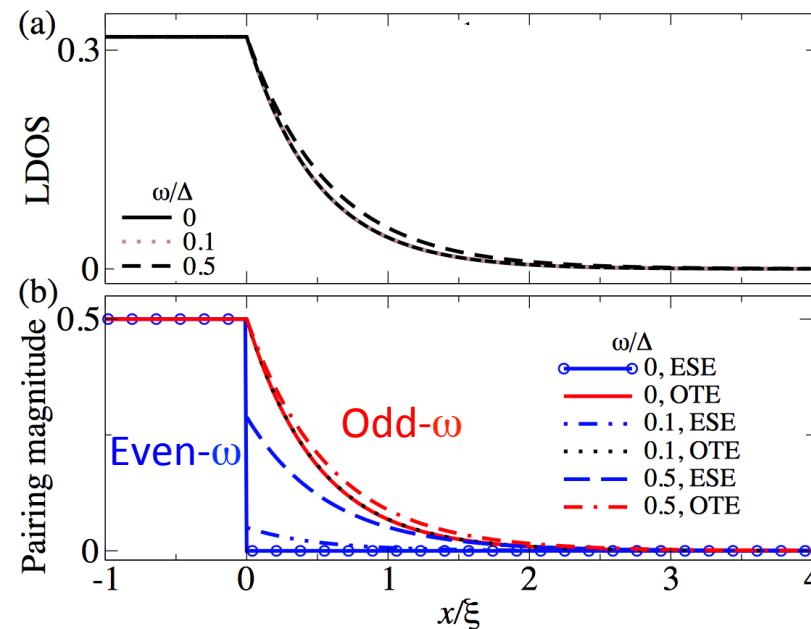
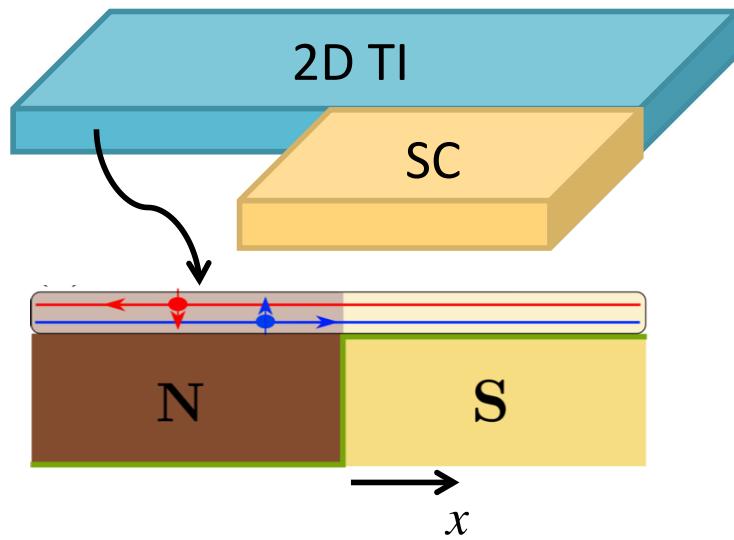
- Dirac spectrum
- Momentum locked to spin: $H \sim \mathbf{k} \cdot \boldsymbol{\sigma}$





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Odd- ω Pairing in TI-SC Structures



Interface + $\mathbf{k} \cdot \boldsymbol{\sigma} \rightarrow$ Odd- ω spin-triplet s -wave pairing

- $\sim d\Delta/dx \rightarrow$ Also induced by supercurrent
- Low-energy states in S

ABS and Balatsky, PRB 86, 144506 (2012), Cayao and ABS, PRB 96, 155426 (2017)



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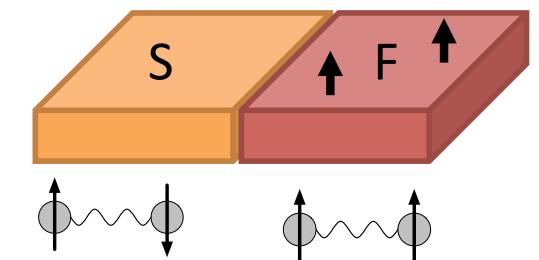
Summary – Intro to Odd- ω SC

- Dynamic & hidden pair correlations

$$\Delta \sim F = \langle \psi_\alpha(t) \psi_\beta(0) \rangle$$

- Well-established in SF heterostructures

- Odd- ω spin-triplet *s*-wave symmetry
 - Long-range proximity effect in F



- Also in SN and S-TI heterostructures

- No magnetism, no time-reversal symmetry breaking
 - Spin-triplet *s*-wave symmetry if SOC



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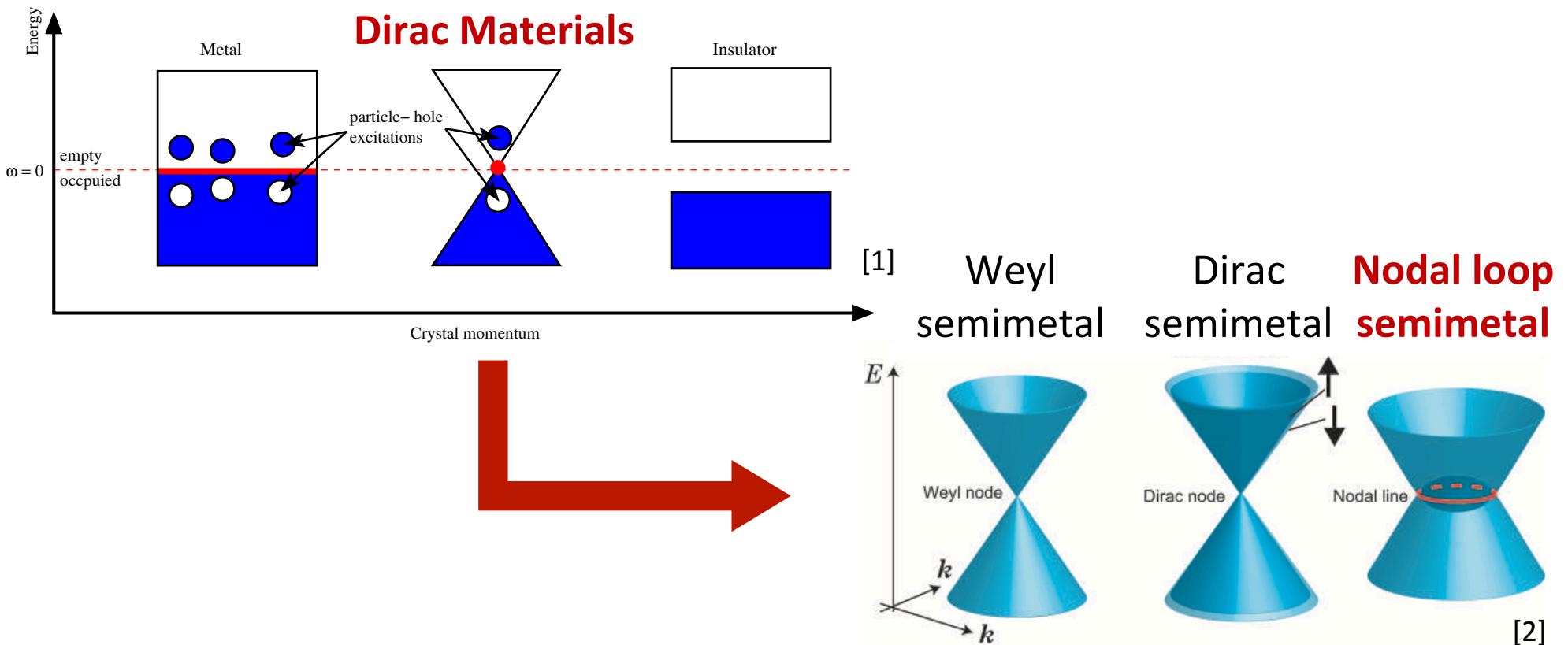
Weyl Nodal Loop Semimetals

- Optimal odd- ω Josephson junctions



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Fermi Surfaces

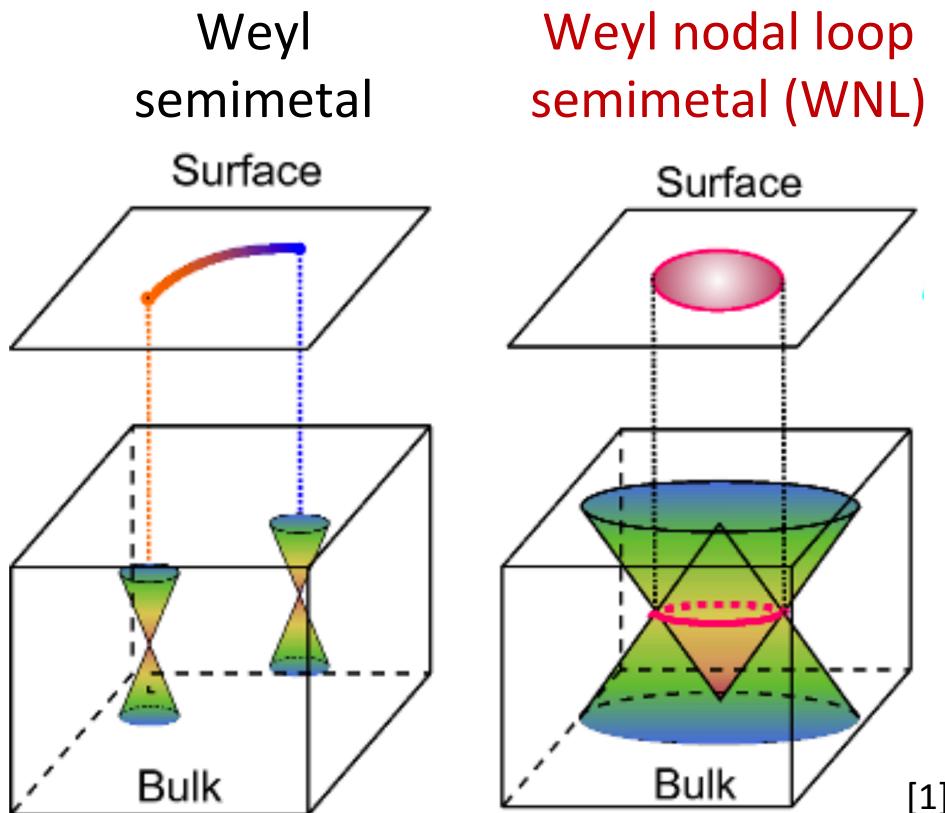


[1]: Wehling, ABS, Balatsky, Adv. Phys. 63, 1 (2014), [2]: Hirayama et al, JPSP 87, 041002 (2018)

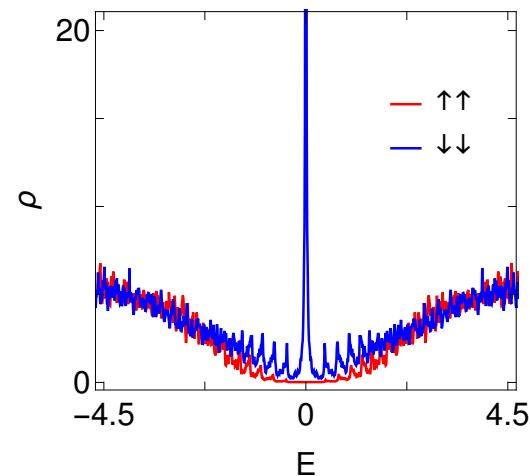


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Surface States



DOS on top surface



WNL has spin-polarized flat
drumhead surface states

Proposed materials:
 $HgCr_2Se_4$, $PbTaSe_2$, ... [2]

[1]: Li et al, Sci. China Matter 62, 23 (2018), [2]: Chen et al, PRL 121, 166802 (2018); Bian et al, Nat. Commun. 7, 10556 (2016)



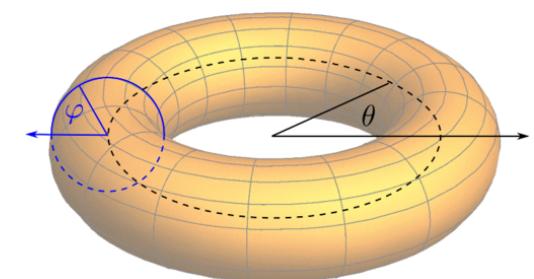
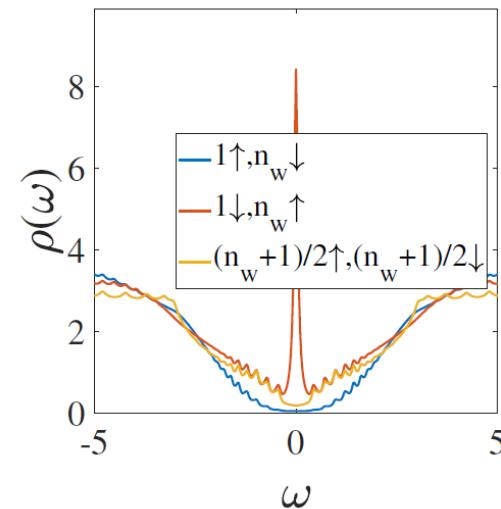
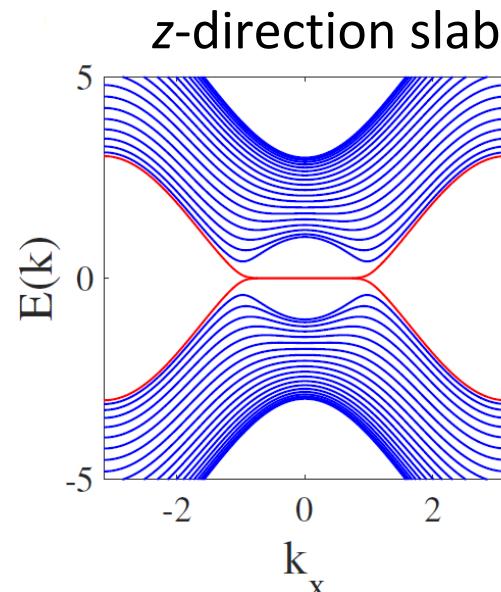
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Weyl Nodal Loop Semimetal

$$H = t_w \left[\sigma_x \left(6 - \alpha_1 - 2 \cos(k_x) - 2 \cos(k_y) - 2 \cos(k_z) \right) + 2 \alpha_2 \sin(k_z) \sigma_y \right] - \mu$$

σ in spin-space \rightarrow **SOC in bulk & spin-polarized DSS**

α sets shape of Fermi surface



Line or donut
Fermi surface

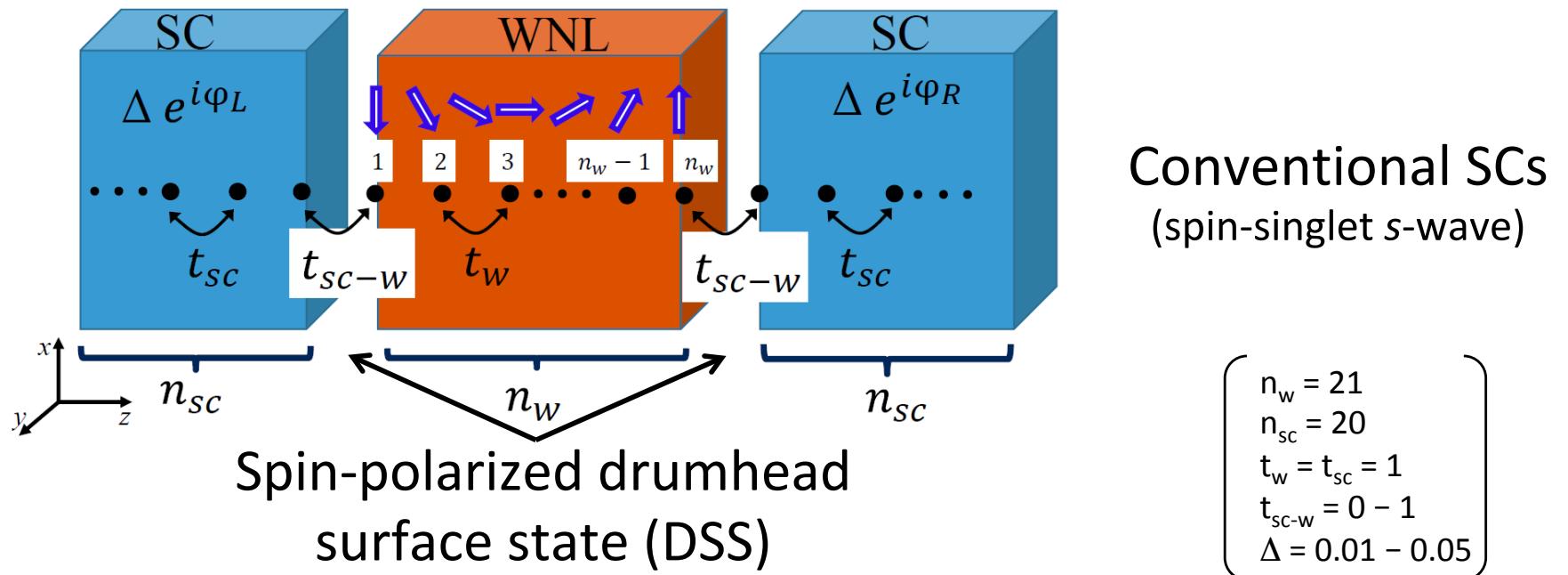


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Proximity-Induced Superconductivity

Fully spin-polarized surface → No spin-singlet pairing

→ No Josephson effect with conventional SCs ?



Parhizgar and ABS, arXiv:1810.09687



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Superconducting Pairing

Green's functions

$$G = (\omega + i0^+ - H)^{-1}$$

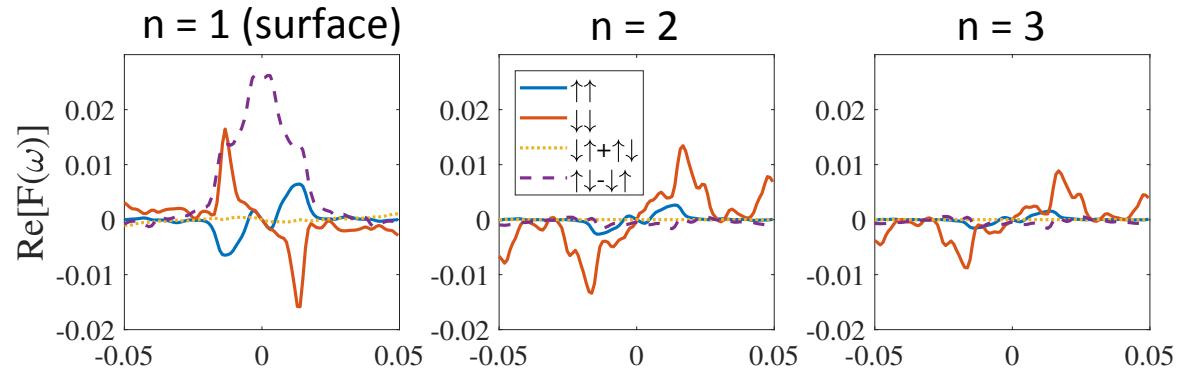
$$G = \begin{pmatrix} g & F \\ \bar{F} & \bar{g} \end{pmatrix}$$



Pair amplitudes:

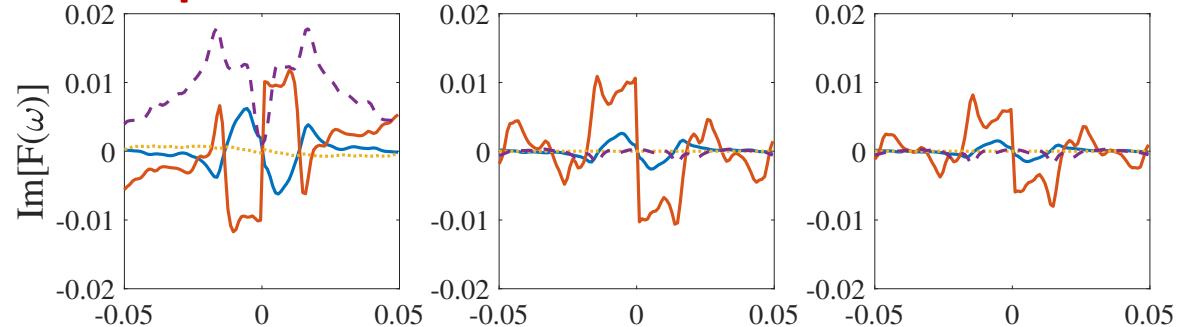
- Singlet s-wave
- Odd- ω triplet s-wave
- Negligible p -waves

s-wave pair amplitudes



Only odd- ω equal-spin triplet s-wave survives

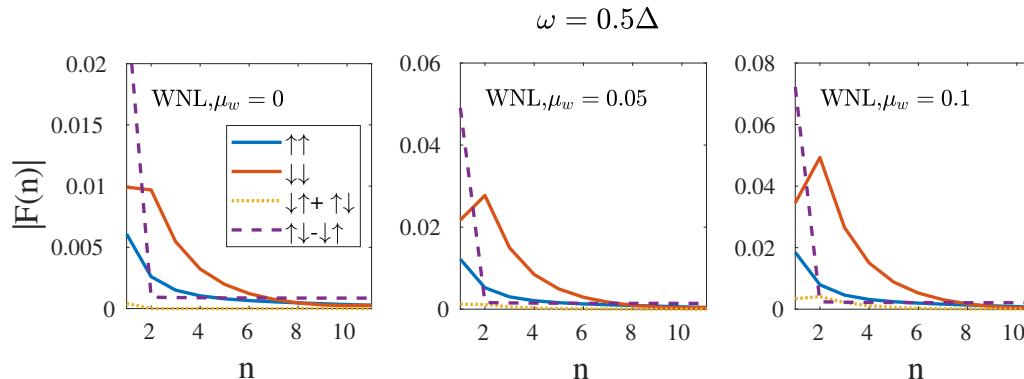
Surface spin-polarization and bulk spin-orbit coupling



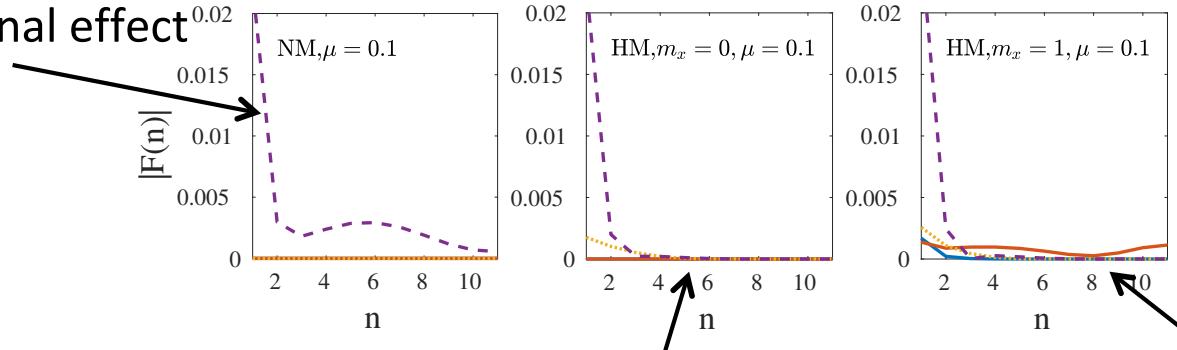


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Pairing in WNL, NM, and HMs



Normal metal (NM):
Conventional effect



Half-metal (HM):
No proximity effect

WNL: Odd- ω
triplet pairing

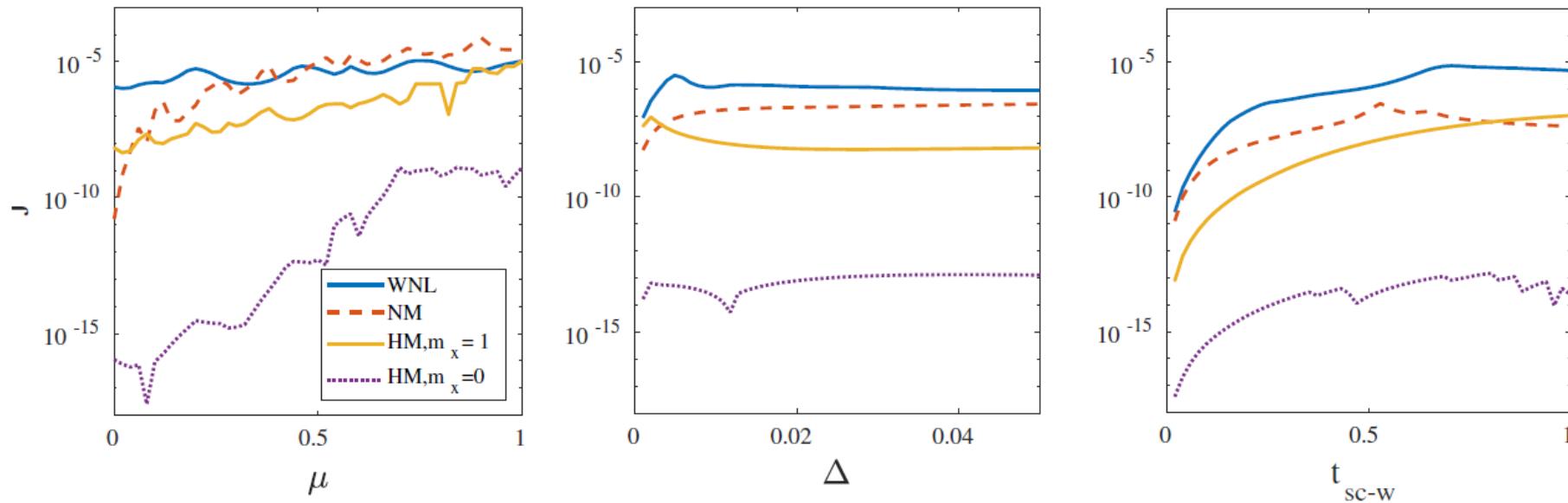
- Independent on μ
- Larger than NM, HMs

M+spin-active interface:
Odd- ω triplet

Josephson Current



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WNL: Huge current ($>$ NM, HM junctions) due to
odd- ω pairing and **flat drumhead surface states**

Parhizgar and ABS, arXiv:1810.09687



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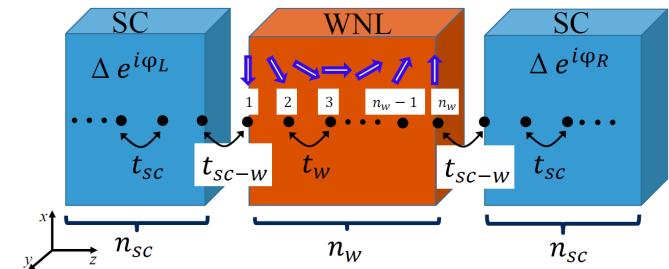
Summary – Odd- ω SC in WNLs

Weyl nodal loop semimetals Josephson junctions

- Spin-polarized flat drumhead surface states (DSS)
- No conventional superconductivity

Josephson junctions

- Pure odd- ω equal-spin triplet superconductivity
- Huge Josephson current, due to odd- ω pairing and DSS



WNLs make optimal odd- ω Josephson junctions

Parhizgar and ABS, arXiv:1810.09687 & Dutta and ABS, arXiv:1902.10014



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Odd- ω Bulk Superconductivity in Multiband Systems

- Simple two-band/orbital superconductors
- Odd- ω pairing and Kerr effect in Sr_2RuO_4
- Dominating odd- ω pairing in doped Bi_2Se_3

Multiband Superconductors



Odd- ω , spin-singlet, *s*-wave pairing **in bulk!**

Multiple bands (orbitals) \rightarrow add band index

$$\Delta \sim \langle c_{\alpha}^{\dagger}(t)c_{\beta}^{\dagger}(0) \rangle$$

Fermi-Dirac statistics $\left[\begin{array}{c} \text{odd-}\omega, \text{ odd-band, spin-singlet, } s\text{-wave} \\ \omega \leftrightarrow \text{band} \qquad \uparrow\downarrow + \downarrow\uparrow \qquad + \end{array} \right]$



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Multiband Superconductors

- S: Spin (even: spin-triplet; odd: spin-singlet)
- P: Spatial parity (even: s,d -wave; odd: p,f -wave)
- O: Orbital or band parity (even; odd orbital)
- T: Time (even; odd-frequency)

SPOT = -1

S = 0	P	T	O	S = 1	P	T	O
even- ω	+	+	+	even- ω	-	+	+
even- ω	-	+	-	even- ω	+	+	-
odd- ω	+	-	-	odd- ω	+	-	+
odd- ω	-	-	+	odd- ω	-	-	-

ABS and Balatsky, PRB 88, 104514 (2013)

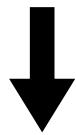


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Simple Two-Band Superconductor

$$H_{ab} = \sum_{k\sigma} \varepsilon_a(k) a_{k\sigma}^\dagger a_{k\sigma} + \varepsilon_b(k) b_{k\sigma}^\dagger b_{k\sigma} + \sum_k \Delta_a(k) a_{k\uparrow}^\dagger a_{-k\downarrow}^\dagger + \Delta_b(k) b_{k\uparrow}^\dagger b_{-k\downarrow}^\dagger + \text{H.c.}$$

Interband
hybridization



$$H_{cd} = \sum_{k\sigma} \varepsilon_c(k) c_{k\sigma}^\dagger c_{k\sigma} + \varepsilon_d(k) d_{k\sigma}^\dagger d_{k\sigma} + \sum_k \Delta_c(k) c_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger + \Delta_d(k) d_{k\uparrow}^\dagger d_{-k\downarrow}^\dagger + \text{H.c.}$$

Diagonal bands

$$+ \sum_k \Delta_{cd}(k) [c_{k\uparrow}^\dagger d_{-k\downarrow}^\dagger + d_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger] + \text{H.c.}$$

Intraband pairing

Interband pairing

$$\Delta_{cd} = \frac{(\Delta_b - \Delta_a)|\Gamma|}{\sqrt{(\varepsilon_a - \varepsilon_b)^2 + 4|\Gamma|^2}}$$



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Time-Dependent Pairing

Time-ordered spin-singlet *s*-wave interband pairing:

$$F^\pm(\tau) = \frac{1}{2N_k} \sum_k T_\tau \langle c_{-k\downarrow}(\tau) d_{k\uparrow}(0) \pm d_{-k\downarrow}(\tau) c_{k\uparrow}(0) \rangle$$

$F^e = F^+(\tau \rightarrow 0^+)$ Even- ω , even-interband pairing

$$F_\omega^o = \left. \frac{\partial F^-}{\partial \tau} \right|_{\tau \rightarrow 0^+}$$

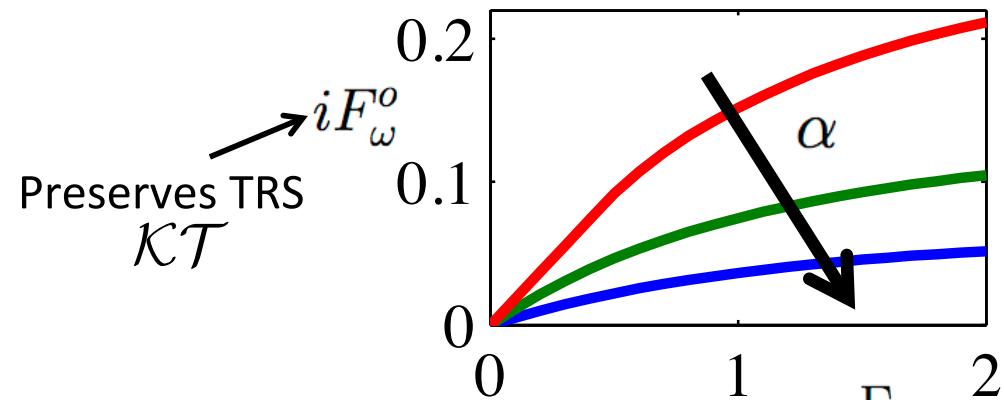
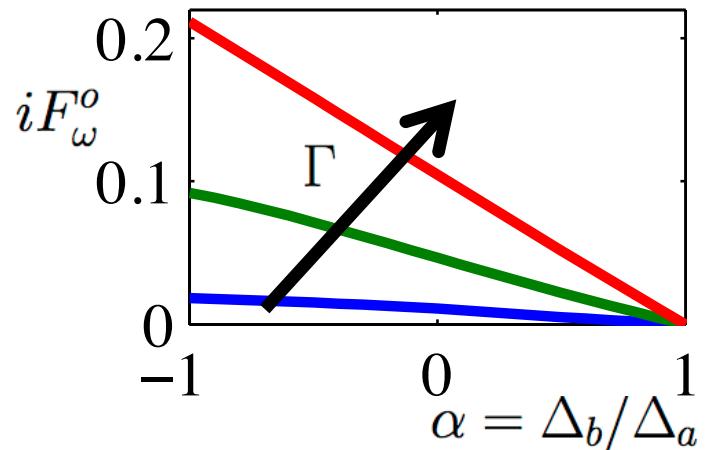
Odd- ω , odd-interband pairing



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Odd- ω , Odd-Interband Pairing

$$\begin{array}{l} \left\{ \begin{array}{l} \varepsilon_a = \varepsilon_b \\ \Delta_a = -\Delta_b \end{array} \right. \xrightarrow{\quad} \left\{ \begin{array}{l} \varepsilon_{c,d} = \varepsilon_a \mp \Gamma \\ \Delta_c = \Delta_d = 0 \\ \Delta_{cd} = \Delta_a \text{ Interband pairing} \end{array} \right. \xrightarrow{\Gamma < \Delta_a} \left\{ \begin{array}{l} F^e = -\frac{1}{2N_k} \sum_k \frac{\Delta_a}{\sqrt{\varepsilon_a^2 + |\Delta_a|^2}} \\ \text{BCS equation} \\ F_\omega^o = i\Gamma F^e \quad \text{Odd-}\omega \end{array} \right. \end{array}$$



ABS and Balatsky, PRB 88, 104514 (2013)

“Band” symmetry breaking

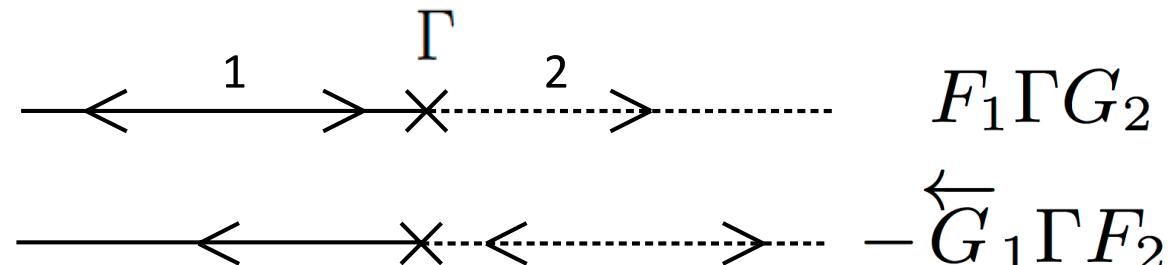


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Using Perturbation Theory

$$H = \sum_{k\sigma} \varepsilon_1(k) a_{k\sigma}^\dagger a_{k\sigma} + \varepsilon_2(k) b_{k\sigma}^\dagger b_{k\sigma} + \boxed{\sum_{k\sigma} \Gamma(k) a_{k\sigma}^\dagger b_{k\sigma} + \text{H.c.}}$$
$$+ \sum_k \Delta_1(k) a_{k\uparrow}^\dagger a_{-k\downarrow}^\dagger + \Delta_2(k) b_{k\uparrow}^\dagger b_{-k\downarrow}^\dagger + \text{H.c.}$$

Interband pairing F_{12} :



Interband Pairing



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Perturbation theory to infinite order in Γ

Odd-interband: $F_{12}^{\text{odd}}(k, i\omega) = \frac{F_{12} - F_{21}}{2} = i\cancel{\omega}\Gamma(\Delta_1 - \Delta_2)/D$

Even-interband: $F_{12}^{\text{even}}(k, i\omega) = \frac{F_{12} + F_{21}}{2} = \Gamma(\Delta_1\varepsilon_2 - \Delta_2\varepsilon_1)/D$

$$\left. \begin{array}{l} D = (\omega^2 + E_1^2)(\omega^2 + E_2^2) - \Gamma^2[2(\varepsilon_1\varepsilon_2 - \omega^2) - \Delta_2^*\Delta_1 - \Delta_1^*\Delta_2] + \Gamma^4 \\ E_j^2 = \varepsilon_j^2 + |\Delta_j|^2 \end{array} \right)$$

Odd- ω pairing: $\Gamma \neq 0, \Delta_1 \neq \Delta_2$

Hybridization and band symmetry breaking

Komendova, Balatsky, and ABS, PRB 92, 04517 (2015)



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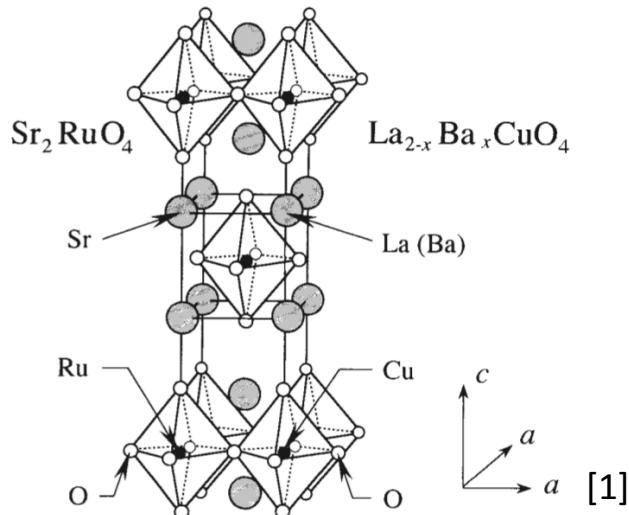
Odd- ω Superconductivity in Sr_2RuO_4

- Introduction to Sr_2RuO_4
- Two- and three-orbital models
- Odd- ω pairing measured by Kerr effect

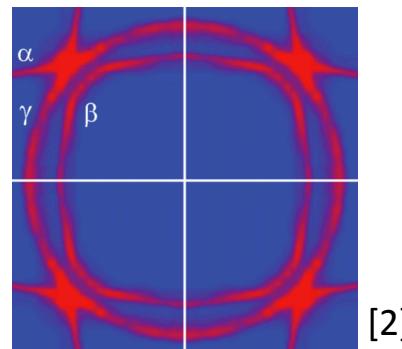


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Strontium Ruthenate, Sr_2RuO_4



Fermi surfaces



Three Ru 4d orbitals:

- $xy \rightarrow \gamma$ (electron-like)
- $xz, yz \rightarrow \beta$ (electron-like) and α (hole-like)

- Non- s -wave (disorder sensitive)
- Spin-triplet (neutron scattering, Knight shift)
- Breaks time-reversal symmetry (Kerr effect)

}

Spin-triplet chiral p_x+ip_y -wave
 $\mathbf{d}(\mathbf{k}) = (0, 0, k_x \pm ik_y)$

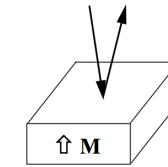
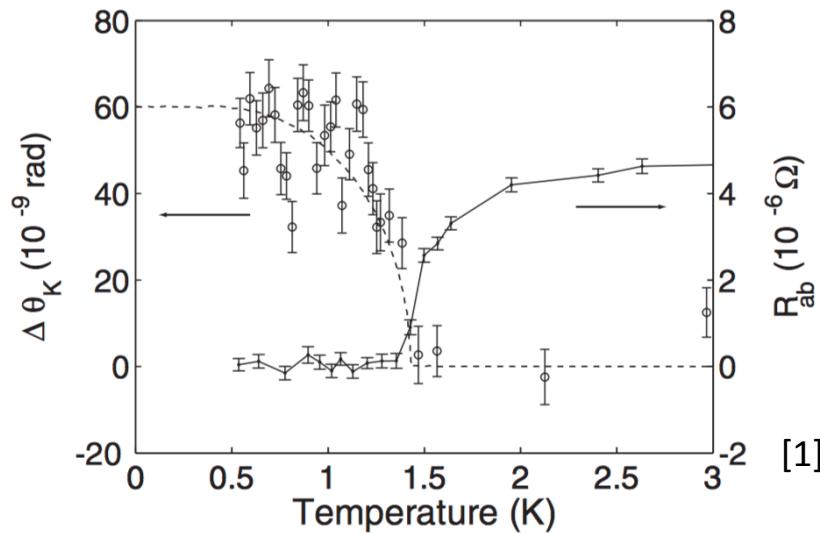
[1]: Maeno et al, Nature 372, 532 (1994), [2]: Damascelli et al, PRL 85, 5194 (2000)



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Kerr Effect in Sr_2RuO_4

Reflected light has a slightly rotated plane of polarization if material breaks time-reversal symmetry (TRS)



SC state in Sr_2RuO_4 breaks TRS

But ...

Clean **single-band** chiral SC has zero
Kerr effect

→ Interband pairing with relative
superconducting phases [2]

Electric-field driven interband transitions with relative SC phases
→ finite transverse Hall current response → finite Kerr effect

[1]: Xia et al, PRL 97, 167002 (2006), [2]: Taylor and Kallin, PRL 108, 157001 (2012); Wysokinski et al, PRL 108, 077004 (2012)



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Two-Orbital Model for Sr_2RuO_4

Hamiltonian $\sum_{\mathbf{k}} \Psi_{\mathbf{k}}^{\dagger} \hat{H}_{\mathbf{k}} \Psi_{\mathbf{k}}$ with $\hat{H}_{\mathbf{k}} = \begin{pmatrix} \hat{H}_0(\mathbf{k}) & \check{\Delta}(\mathbf{k}) \\ \check{\Delta}^{\dagger}(\mathbf{k}) & -\hat{H}_0(-\mathbf{k}) \end{pmatrix}$

- xz (1), yz (2) orbitals
 $\rightarrow \alpha, \beta$ bands
- Spin-triplet $\mathbf{d} \parallel \hat{\mathbf{z}}$

$$\Psi_{\mathbf{k}}^{\dagger} = (c_{\mathbf{k}\uparrow 1}^{\dagger} c_{\mathbf{k}\uparrow 2}^{\dagger} c_{-\mathbf{k}\downarrow 1} c_{-\mathbf{k}\downarrow 2})$$

$$\hat{H}_0(\mathbf{k}) = \begin{pmatrix} \xi_1 & \epsilon_{12} \\ \epsilon_{12} & \xi_2 \end{pmatrix}, \quad \check{\Delta}(\mathbf{k}) = \begin{pmatrix} \Delta_1 & \Delta_{12} \\ \Delta_{12} & \Delta_2 \end{pmatrix}$$

Intraorbital energy

Interorbital hybridization

Interorbital pairing

Intraorbital pairing

Odd- ω Pairing and Kerr Effect



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Odd- ω , odd-interorbital pairing:

$$F_{12} - F_{21} = i\omega[(\Delta_2 - \Delta_1)\epsilon_{12} + \Delta_{12}(\xi_1 - \xi_2)]/D \quad (D \sim \omega^2)$$

Interorbital hybridization Interorbital pairing
+ gap asymmetry + dispersion asymmetry

Kerr effect: [1]

$$\sigma_H \propto \epsilon_{12}\text{Im}(\Delta_1^*\Delta_2) + \xi_1\text{Im}(\Delta_2^*\Delta_{12}) - \xi_2\text{Im}(\Delta_1^*\Delta_{12})$$

Interorbital hybridization Interorbital pairing
+ gap asymmetry + orbital asymmetry

Intrinsic Kerr effect → odd- ω superconductivity

Komendova and ABS, PRL 119, 087001 (2017), [1]: Taylor and Kallin, PRL 108, 157001 (2012)

Generic Three-Orbital Model



$$\hat{H}_0(\mathbf{k}) = \begin{pmatrix} \xi_1 & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{12} & \xi_2 & \epsilon_{23} \\ \epsilon_{13} & \epsilon_{23} & \xi_3 \end{pmatrix}, \quad \check{\Delta}(\mathbf{k}) = \begin{pmatrix} \Delta_1 & \Delta_{12} & \Delta_{13} \\ \Delta_{12} & \Delta_2 & \Delta_{23} \\ \Delta_{13} & \Delta_{23} & \Delta_3 \end{pmatrix}$$

xy (1), xz (2), yz (3) orbitals $\rightarrow \gamma, \alpha, \beta$ bands

General interorbital pairing:

$$F_{AS} = \sum_{i,j,k=1,\dots,N} \epsilon_{ijk} F_{ij}$$

$$F_{AS}(-\omega) = -F_{AS}(\omega)$$

odd- ω , odd-interorbital

$$F_S = \sum_{i \neq j=1,\dots,N} F_{ij}$$

$$F_S(-\omega) = F_S(\omega)$$

even- ω , even-interorbital



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Examples Odd- ω Pairing

- No interorbital pairing and $\epsilon_{ij} = \Gamma$:

$$F_{\text{odd}} = 2\Gamma i\omega [\Delta_1(\epsilon_2 - \epsilon_3)(\epsilon_2 + \epsilon_3 + \Gamma) + |\Delta_1|^2(\Delta_3 - \Delta_2) \\ + \text{two cyclic permutations}] / D_3$$

- Only $\epsilon_{23} \neq 0, \Delta_{23} \neq 0$ ($xz, yz \rightarrow \alpha, \beta$ bands with hybridization, $xy \rightarrow \gamma$ band)

$$F_{\text{odd}} = 2i\omega [\Delta_{23}(\epsilon_3 - \epsilon_2) + \epsilon_{23}(\Delta_3 - \Delta_2)] / D'_3$$

→ **Odd- ω pairing with any interorbital processes**

Exactly same as finite Kerr rotation [1]

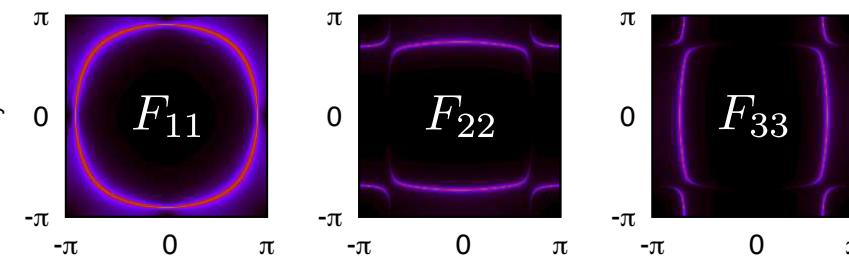


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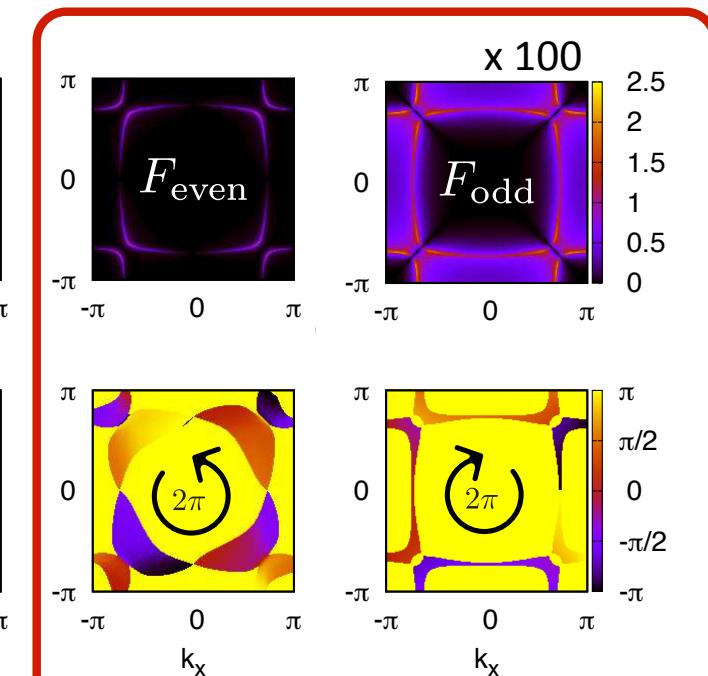
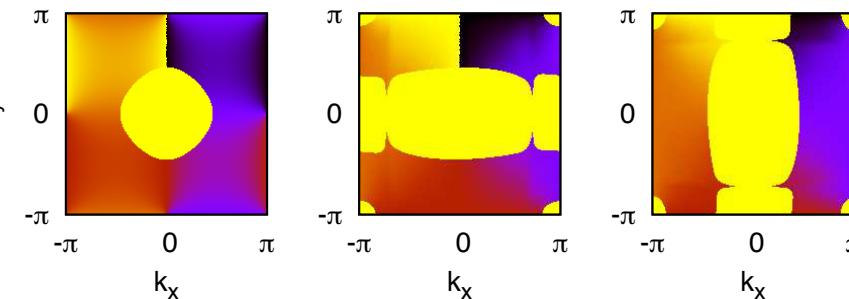
Pair Amplitudes in Sr_2RuO_4

Only $\epsilon_{23} \neq 0, \Delta_{23} \neq 0$

Amplitude

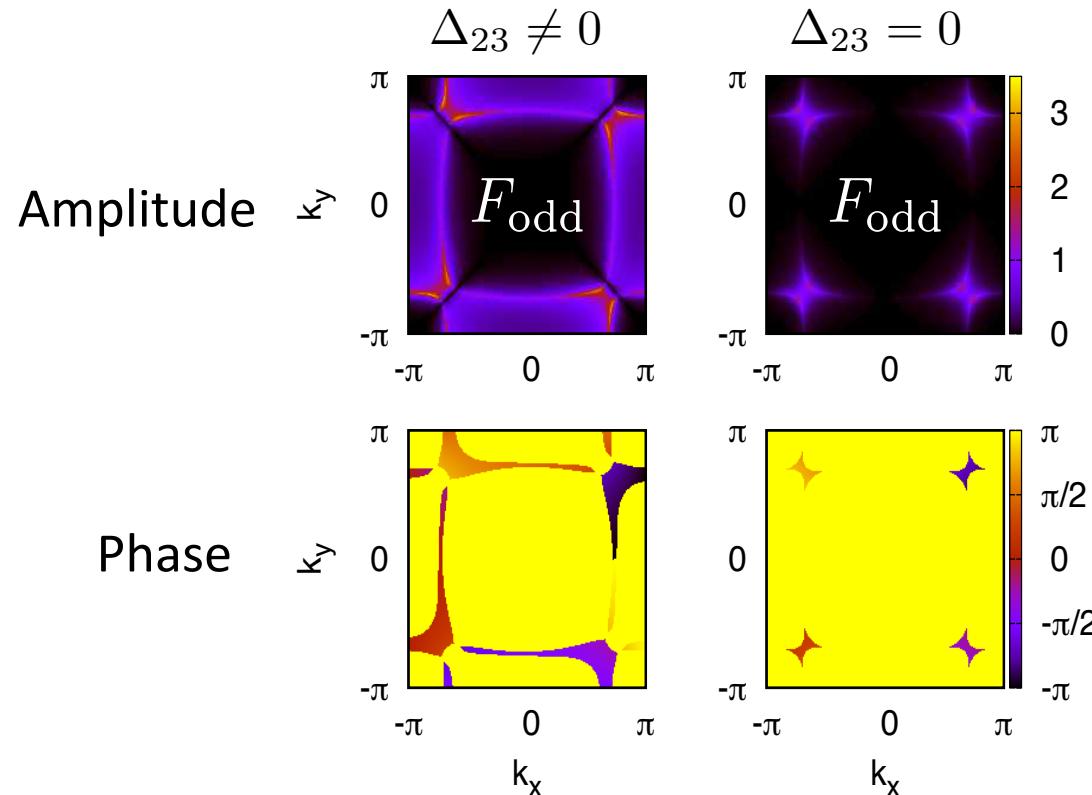


Phase



- α, β bands only
- Chiral p -wave symmetry

Odd- ω Pairing in k-Space



- Odd- ω , odd-interorbital
- Chiral p -wave symmetry
- Peaked at α, β hybridization
- Nodal structure dependent on exact parameters



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Odd- ω Superconductivity in Doped Bi_2Se_3

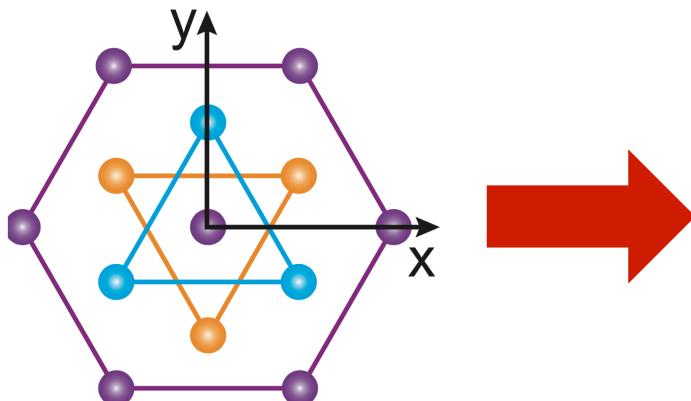
- Nematic order → Dominating odd- ω pairing
- Diamagnetic Meissner effect



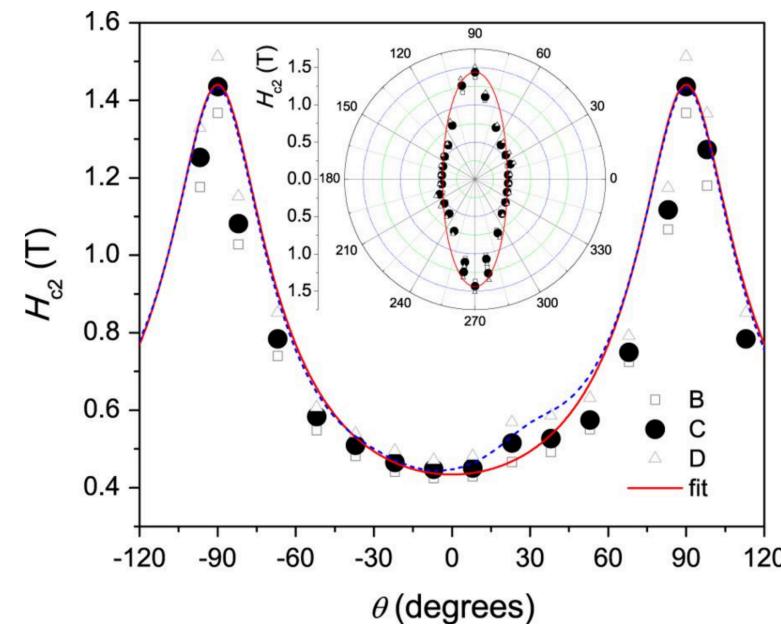
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Doped Topological Insulator Bi_2Se_3

- Cu, Sr, or Nb doping $\text{Bi}_2\text{Se}_3 \rightarrow$ Intrinsic SC state
- Normal state: D_{3d} \rightarrow Two-fold symmetric SC state



Stacked 3-fold symmetric layers
Se-Bi-Se-Bi-Se





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Normal State – Doped Bulk TI

Normal state Hamiltonian:

$$\hat{\mathcal{H}}_0 = m\sigma_x + v(k_x s_y - k_y s_x) \otimes \sigma_z + v_z k_z \sigma_y - \mu$$

Orbital hybridization,
mass gap

Orbital space

Doping
(from Cu, Sr, Nb)

Energy spectrum:

$$\epsilon_{\pm}^0 = \pm \sqrt{m^2 + v^2(k_x^2 + k_y^2) + v_z^2 k_z^2} - \mu$$

Gapped Dirac spectrum

Fu and Berg, PRL 105, 097001 (2010)

Nematic Superconducting State



Assume k -independent order parameter:

- Nematic SC state \rightarrow 2D irreducible representation E_u

$$\Delta_{E_u} : \{c_{1\uparrow}c_{2\uparrow}, c_{1\downarrow}c_{2\downarrow}\}$$

$$(\hat{\Delta}_x, \hat{\Delta}_y) = \Delta(s_0 \otimes i\sigma_y, s_z \otimes i\sigma_y)$$

**Odd-interorbital
spin-triplet *s*-wave**

$$\hat{\Delta} = A_x \hat{\Delta}_x + A_y \hat{\Delta}_y$$

$$(A_x, A_y) = (\cos(\theta), \sin(\theta))$$

Superconducting Pairing

Pairing $(\mathcal{F}^{(1)} * D)$	Spin	Parity	Orbital	Freq.
$2iA_{\pm}m\omega\Delta$	$\uparrow\uparrow, \downarrow\downarrow$	s	intra	odd
$2i(A \times k)v\omega\Delta$	$\uparrow\downarrow - \downarrow\uparrow$	$p_{x,y}$	even-inter	odd
$2A_{\pm}k_zv_z\mu\Delta$	$\uparrow\uparrow, \downarrow\downarrow$	p_z	intra	even
$2(A \cdot k)vm\Delta$	$\uparrow\downarrow + \downarrow\uparrow$	$p_{x,y}$	intra	even
$2(A \times k)vk_zv_z\Delta$	$\uparrow\downarrow - \downarrow\uparrow$	d	intra	even
$iA_{\pm}(m^2 - \mu^2 + \omega^2)\Delta$	$\uparrow\uparrow, \downarrow\downarrow$	s	odd-inter	even
$iA_{\pm}k_z^2v_z^2\Delta$	$\uparrow\uparrow, \downarrow\downarrow$	d	odd-inter	even
$2A_{\pm}k_zv_z\Delta$	$\uparrow\uparrow, \downarrow\downarrow$	p_z	even-inter	even
$2(A \cdot k)v\mu\Delta$	$\uparrow\downarrow + \downarrow\uparrow$	$p_{x,y}$	even-inter	even

$$\left. \begin{aligned} A_{\pm} &= A_x \pm iA_y, A \cdot k = A_x k_x + A_y k_y, A \times k = A_x k_y - A_y k_x \\ D &= \prod_i ((i\omega)^2 - (\epsilon_i^0)^2) \end{aligned} \right\}$$



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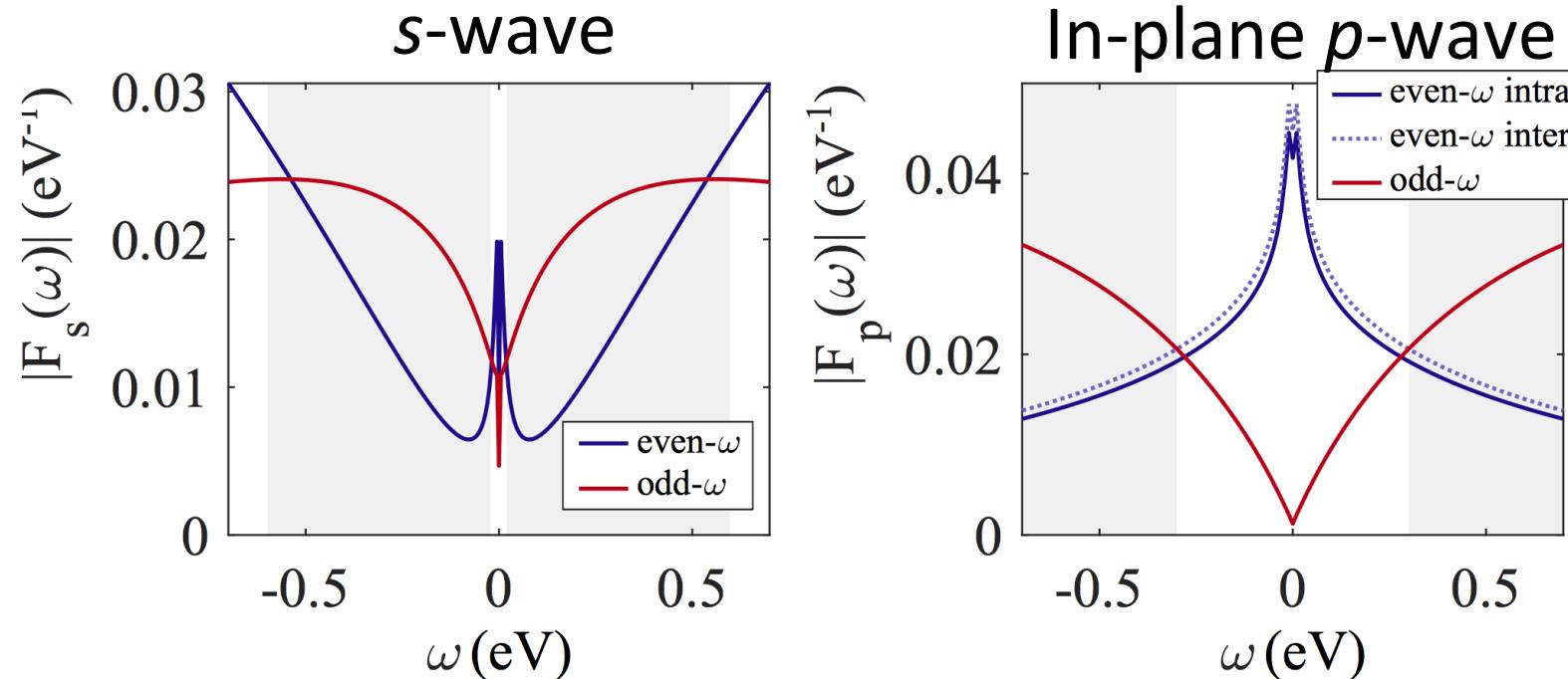
Intra-orbital odd- ω

Generated by interorbital
hybridization m

Assumed order parameter

Schmidt, Parhizgar, ABS, arXiv:1909.02921

Comparing Pair Amplitudes



Dominating odd- ω intra-orbital pairing



Meissner Effect

- Current response to external field

$$\langle j_\mu(\vec{q}, f) \rangle = -K_{\mu\nu}(\vec{q}, f) A_\nu(\vec{q}, f)$$

- Meissner effect: static, uniform field ($\vec{q} \rightarrow 0, f \rightarrow 0$)
- Doped TIs (non-standard behavior)
 - No diamagnetic current & off-diagonal current operator

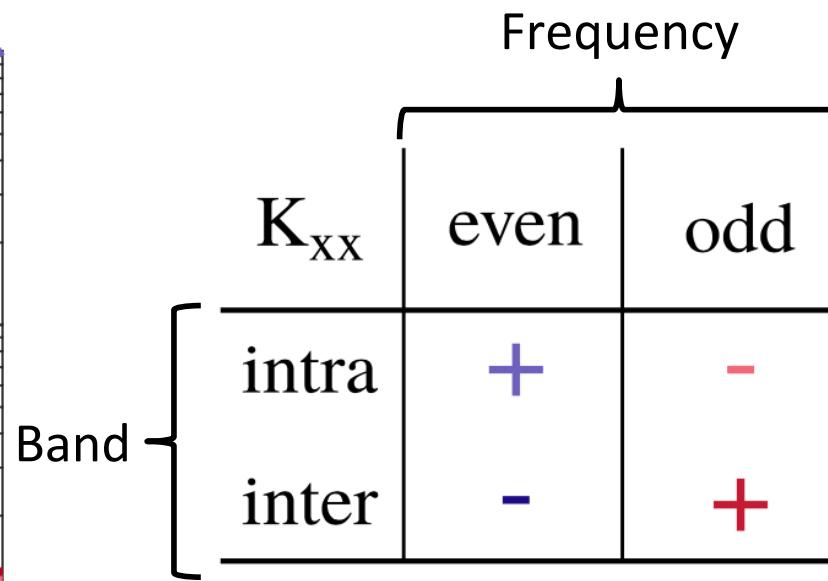
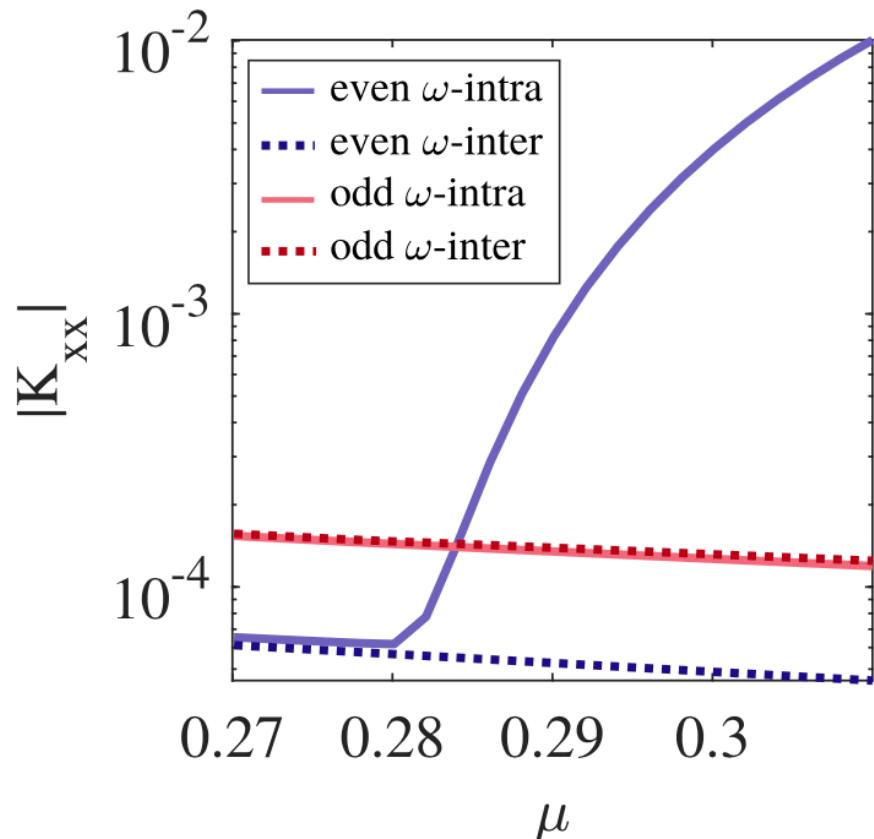
$$\Rightarrow K_{\mu\mu}^{(S)} = \int d\vec{k} T \sum_{\omega_n} \text{Tr} [f^e \bar{j}_\mu \bar{f}^e j_\mu + f^o \bar{j}_\mu \bar{f}^o j_\mu]$$

Even- ω Odd- ω
Usually >0 , diamagnetic Usually <0 , paramagnetic

Meissner Effect



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Diamagnetic Meissner effect

Odd- ω : Interband contributions are large and diamagnetic



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Summary – Multiband Odd- ω SC

- Bulk odd- ω SC in multiband systems
 - Finite interband/orbital pairing or hybridization
- Odd- ω SC in Sr_2RuO_4 measured by Kerr effect
- Dominating odd- ω pairing in doped Bi_2Se_3
 - Odd- ω intraorbital spin-triplet s-wave
 - Diamagnetic Meissner effect due to interband contributions
- Band → layer, wires, dots, valleys, ...
 - Bilayer: Parhizgar and ABS, Sci. Rep. 7, 9817 (2017); Double wires: Ebisu et al, Prog. Theor. Exp. Phys. 083I01 (2016); Double dots: Burset et al, 93, 201402 (2016), Two valleys: Triola et al, PRL 116, 257001 (2016)



Summary

- Dynamic & hidden SC pair correlations
 - $\Delta \sim F = \langle \psi_\alpha(t) \psi_\beta(0) \rangle$
 - Well-established in SF heterostructures
- Weyl nodal loop semimetals as optimal odd- ω Josephson junctions
- Odd- ω SC in multiband systems: **SPOT = -1**
 - Ubiquitous with interorbital/dot/wire/valley processes
 - Measured by Kerr effect in Sr_2RuO_4 and UPt_3
 - Dominates in doped topological insulator Bi_2Se_3

