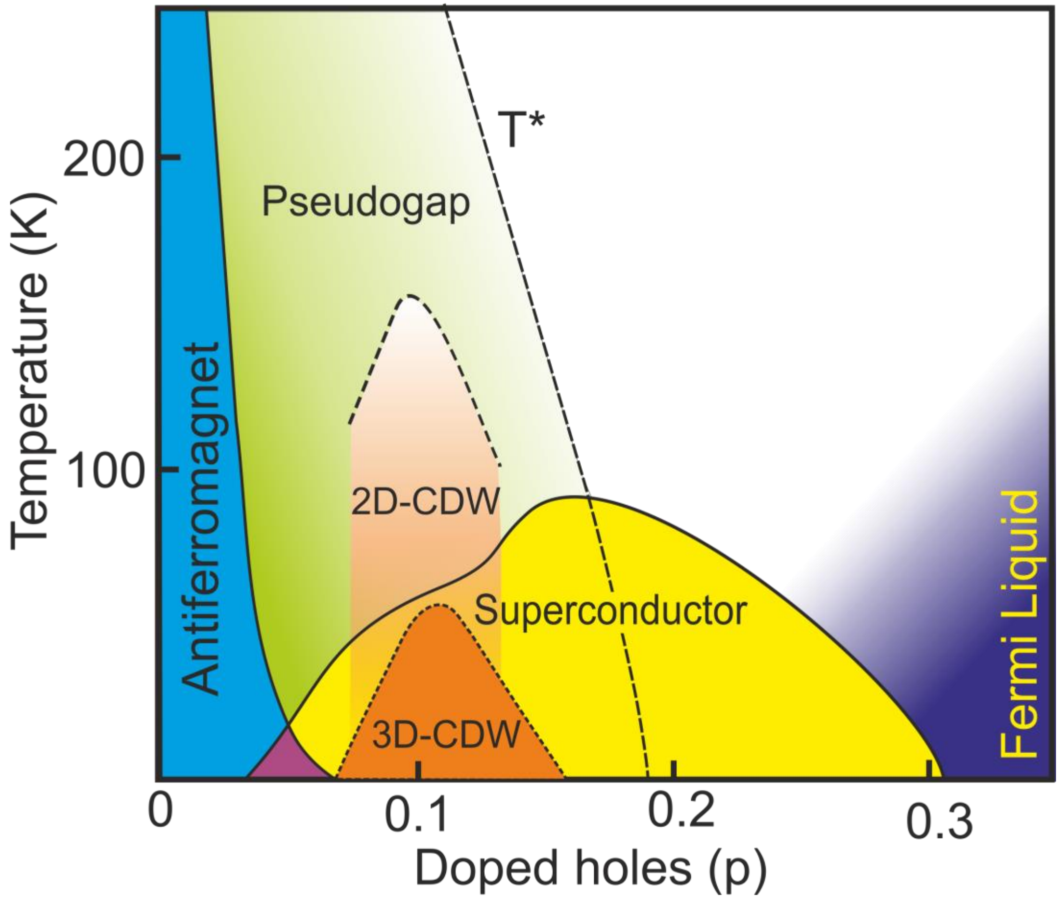
Why study the normal state ?

Superconductivity competes with normal state
Need superconducting state to have lower energy for $T<T_c$
So need to understand details of normal state

- Competing phases
- Anomalous scattering
- Quasiparticle / coherence / collective excitations

Bulk transport and thermodynamic properties

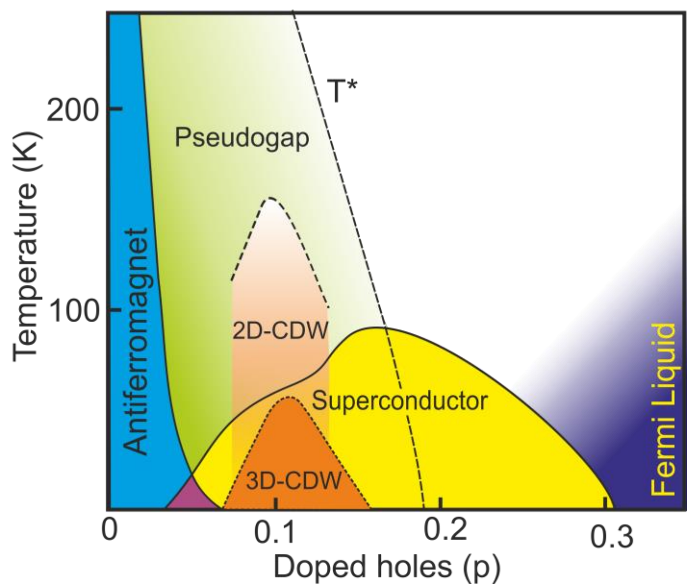
1. Phase diagram
2. Electronic structure
3. Resistivity
4. Hall Effect
5. Boltzmann theory
6. Quantum Oscillations



Cuprates: Materials

Hg-Family	Abbrev.	T_c
$HgBa_2Ca_3Cu_4O_{10+\delta}$	Hg-1234	125 K
$HgBa_2Ca_2Cu_3O_{8+\delta}$	Hg-1223	134 K (164 K @ 30GPa) – highest T_c
$HgBa_2CuO_{4+\delta}$	Hg-1201	95 K
Tl-Family		
$Tl_2Ba_2Ca_2Cu_3O_{10+\delta}$	Tl-2223	128 K
$Tl_2Ba_2CaCu_2O_{6+\delta}$	Tl-2212	118 K
$Tl_2Ba_2CuO_{6+\delta}$	Tl-2201	95 K (can be highly overdoped)
$TlBa_2Ca_3Cu_4O_{11+\delta}$	Tl-1234	112 K
$TlBa_2Ca_2Cu_3O_{9+\delta}$	Tl-1223	120 K
$TlBa_2CaCu_2O_{7+\delta}$	Tl-1212	103 K
Bi-Family		
$Bi_2Sr_2Ca_2Cu_3O_{10+\delta}$	Bi-2223	110 K
$Bi_2Sr_2CaCu_2O_{8+\delta}$	Bi-2212	91 K (photoemission/tunneling –cleaves)
$Bi_2Sr_2CaCu_2O_{8+\delta}$	Bi-2201	35 K
Y-Family		
$YBa_2Cu_3O_{7+\delta}$	Y-123	94 K (clean – most highly studied)
$Y_2BaCu_4O_{7+\delta}$	Y-124	82 K
La-Family		
$La_{2-x}Sr_xCuO_{4+\delta}$	LaSr-214	40 K (full doping range)
$La_{2-x}Ba_xCuO_{4+\delta}$	LaBa-214	30 K (1st cuprate superconductor)
Others		
$Ca_{1-x}Sr_xCuO_2$		110 K
$Nd_{2-x}Ce_xCuO_{4+\delta}$		30K

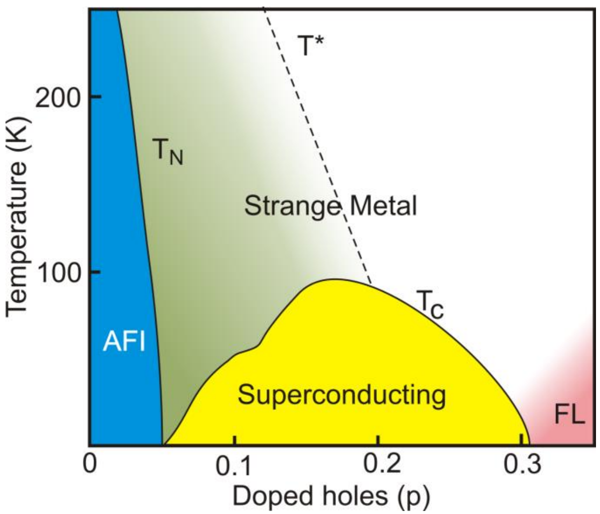
Hole doped



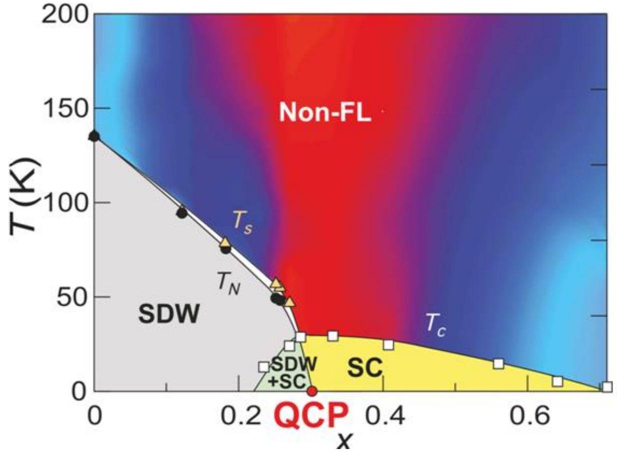
Electron doped

Superconductors meditated by magnetic interactions ?

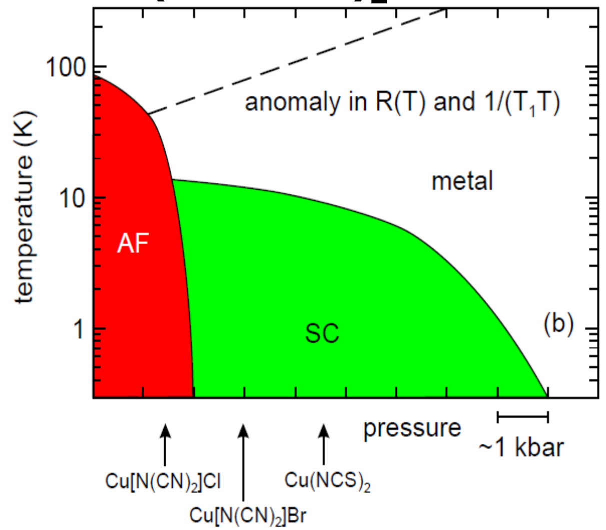
Cuprate



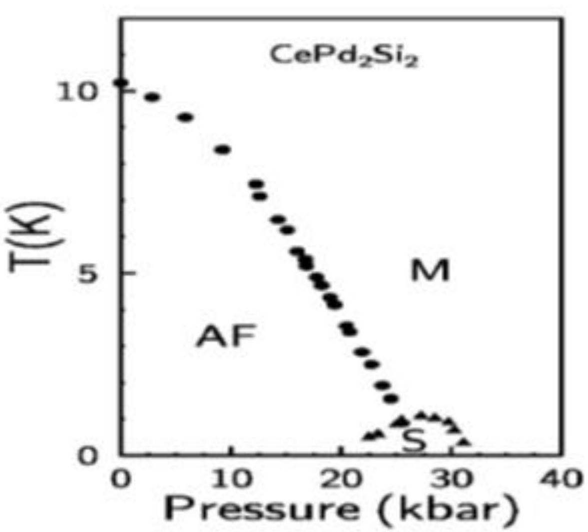
Iron based (pnictide)



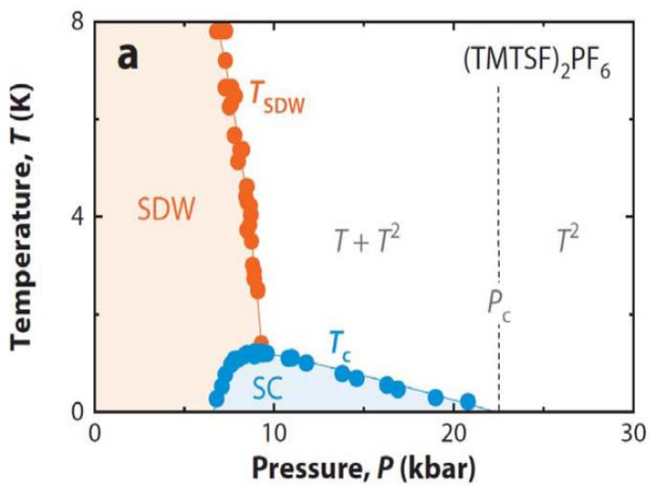
κ–(BEDTTF)$_2$X



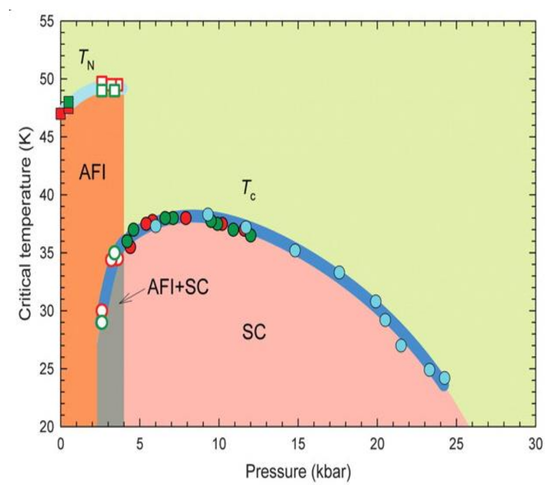
Heavy Fermion



Bechgaard salts

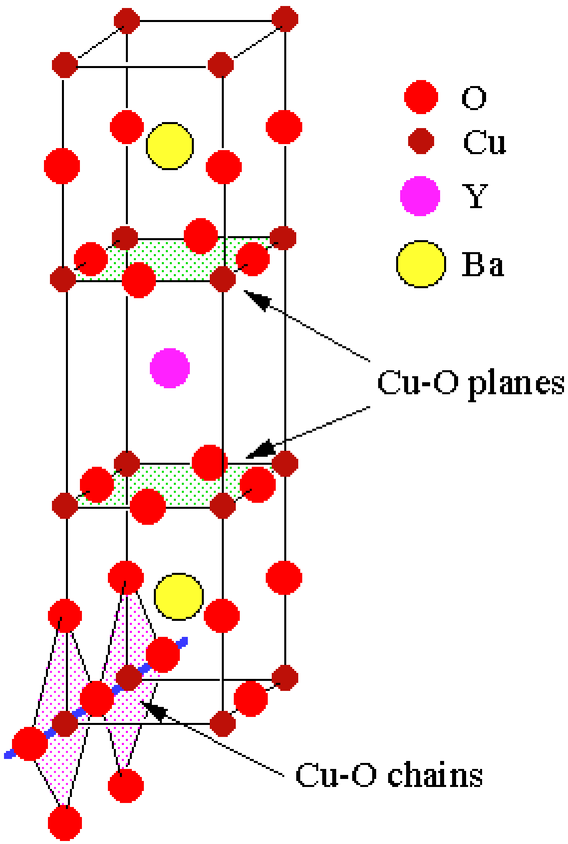


Cs$_3$C$_{60}$

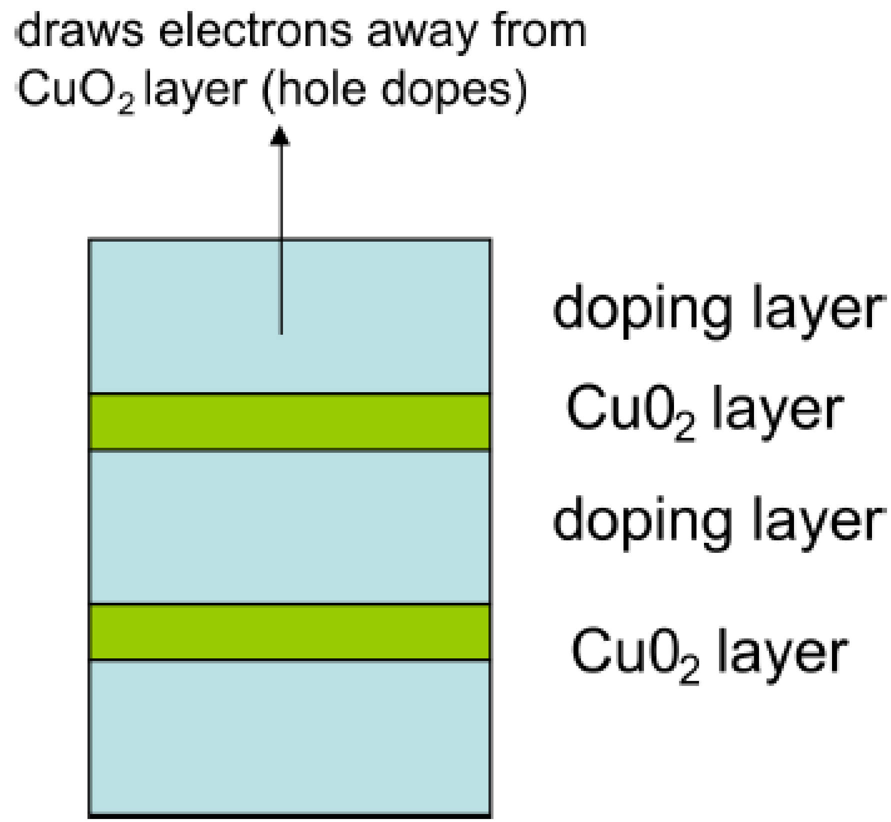


Cuprates : Basic structure

$YBa_2Cu_3O_{7-\delta}$



Two dimensional copper-oxides



Electronic structure : Parent compound La_2CuO_4

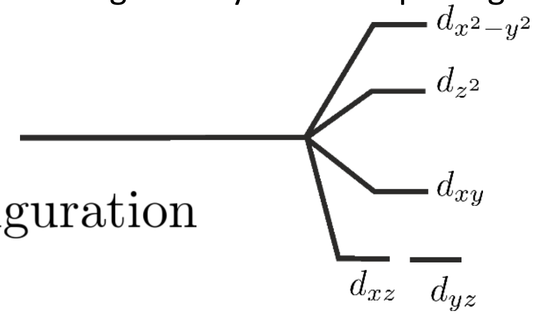
La_2CuO_4

Formal valence counting

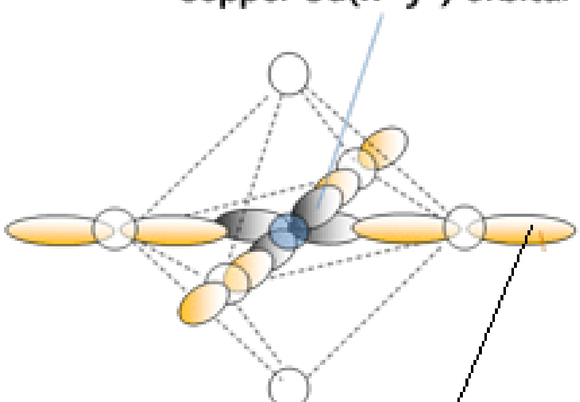
$La_2^{(3+)}Cu^{(2+)}O_4^{(2-)}$

Cu^{2+} is in $3d^9$ configuration

Tetragonal crystal field splitting

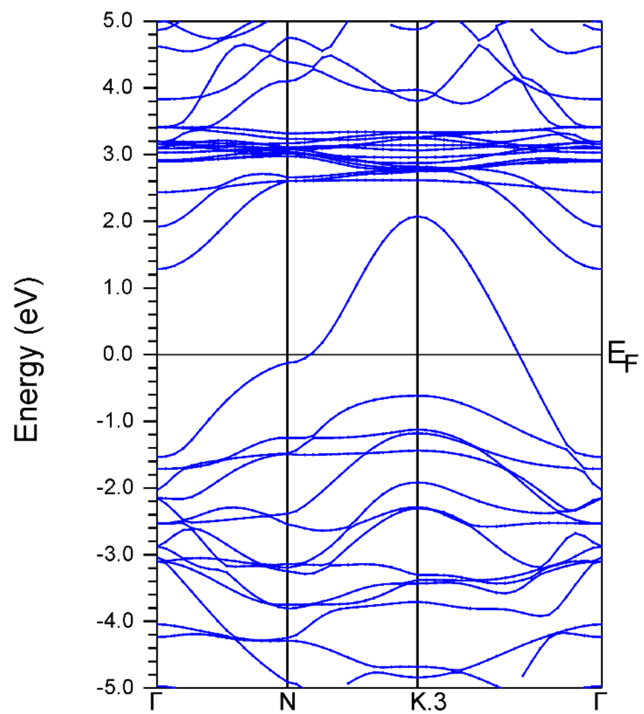


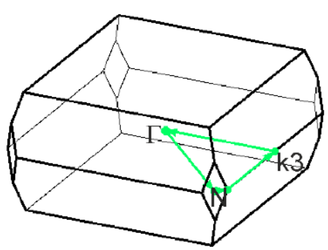
- Single unpaired electron on $3d$ orbitals.
- Cu is in tetragonal structure: leads to Crystal field splitting of d orbitals
- Highest energy orbital which contains a hole is $d_{x^2-y^2}$ orbital
- Cu $3d_{x^2-y^2}$ level strongly hybridises with O $2p$



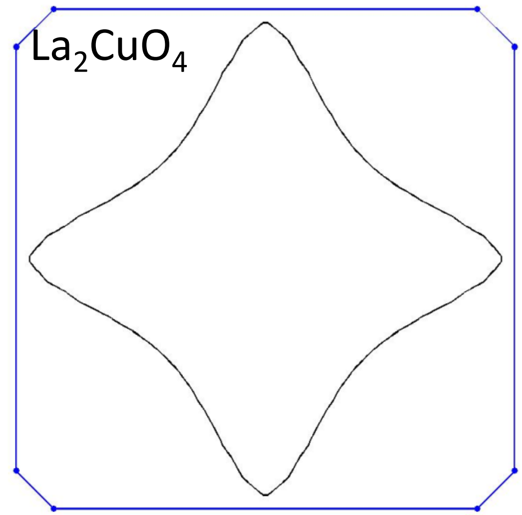
Density functional theory band-structure : DFT-LDA : La_2CuO_4

Single hole band
from Cu and O orbitals



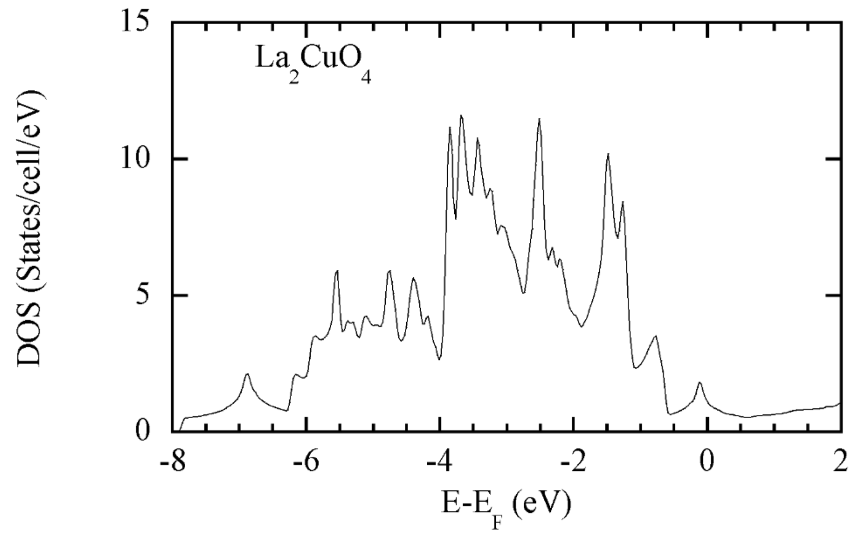


Fermi surface



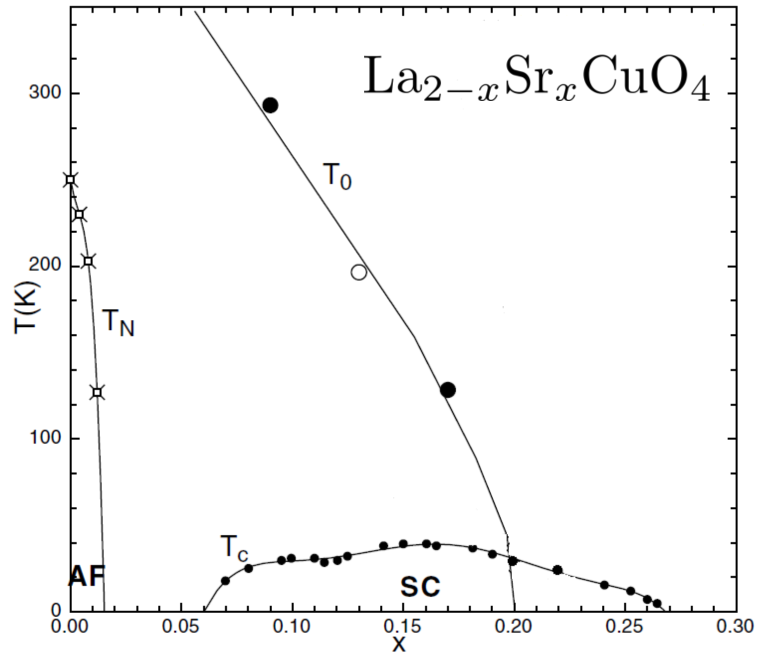
DFT-LDA predicts that
La_2CuO_4 is half filled
i.e., 1 electron per unit cell

Fermi surface almost 2D
Weak c-axis dispersion



La_2CuO_4 Mott Physics

DFT-LDA is wrong !

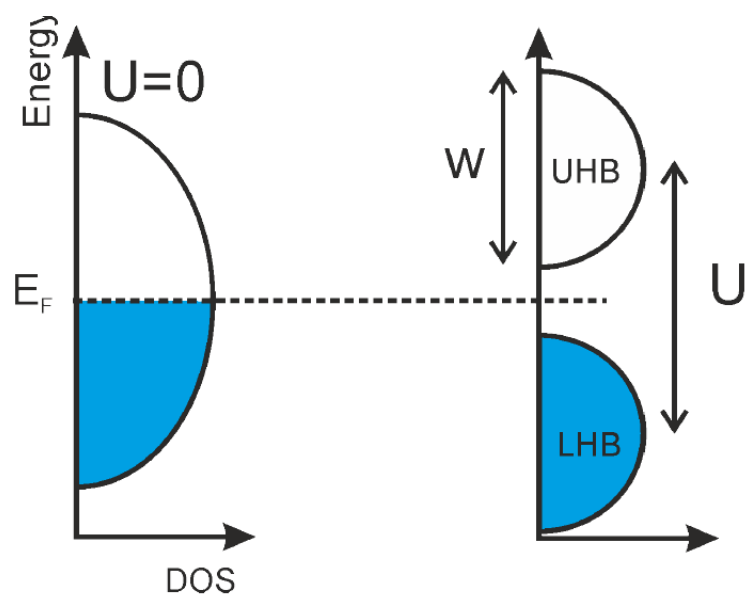


Undoped La_2Cu0_4 is an antiferromagnetic insulator

Gap formed at the Fermi level because of electronic correlations (fluctuating U) not included in LDA band-structure

Hubbard Model

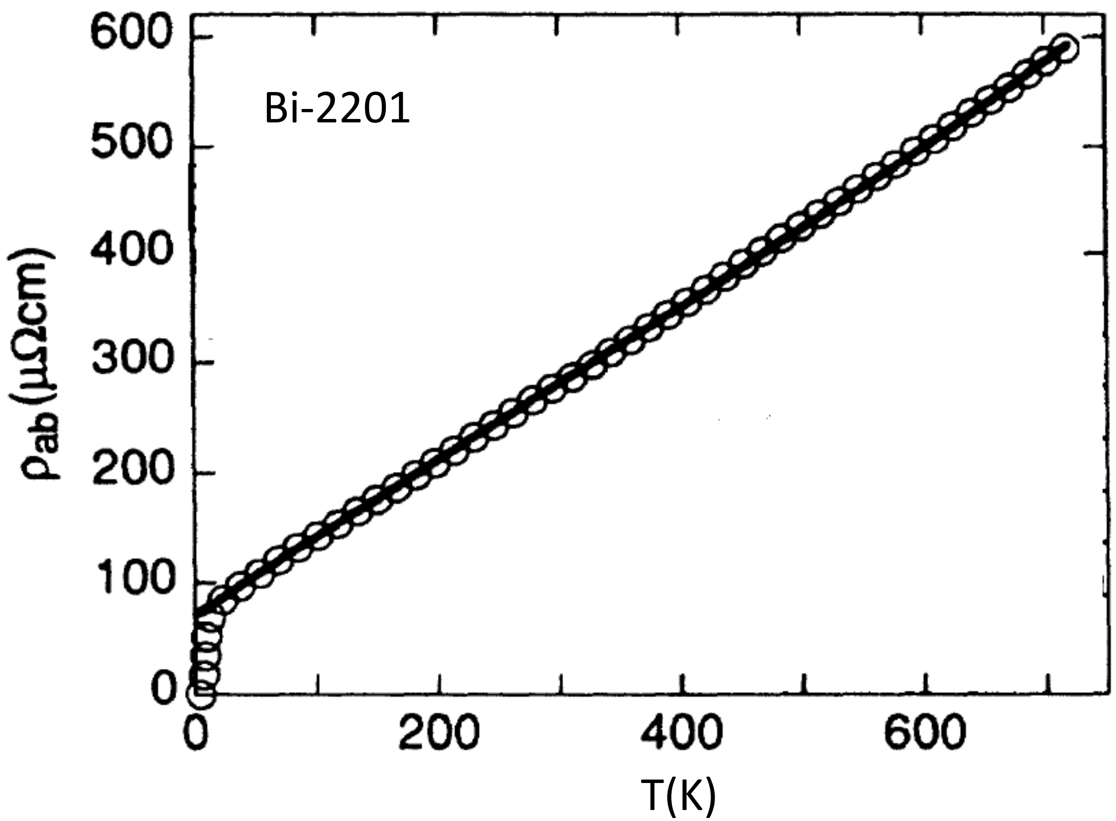
$$\mathcal{H} = -\sum_{i,j,\sigma} t_{ij}(c^{\dagger}_{i\sigma} c_{j\sigma} + h.c.) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$



How does gap close ?
How much 'Mottness' remains in metallic state?

Transport properties in the metallic state

Normal state properties: Linear resistivity



Resistivity linear with T over an anomalously large range of temperature

C.f. Simple metals where

$$\rho \propto T \text{ for } T \gtrsim \theta_D/3$$

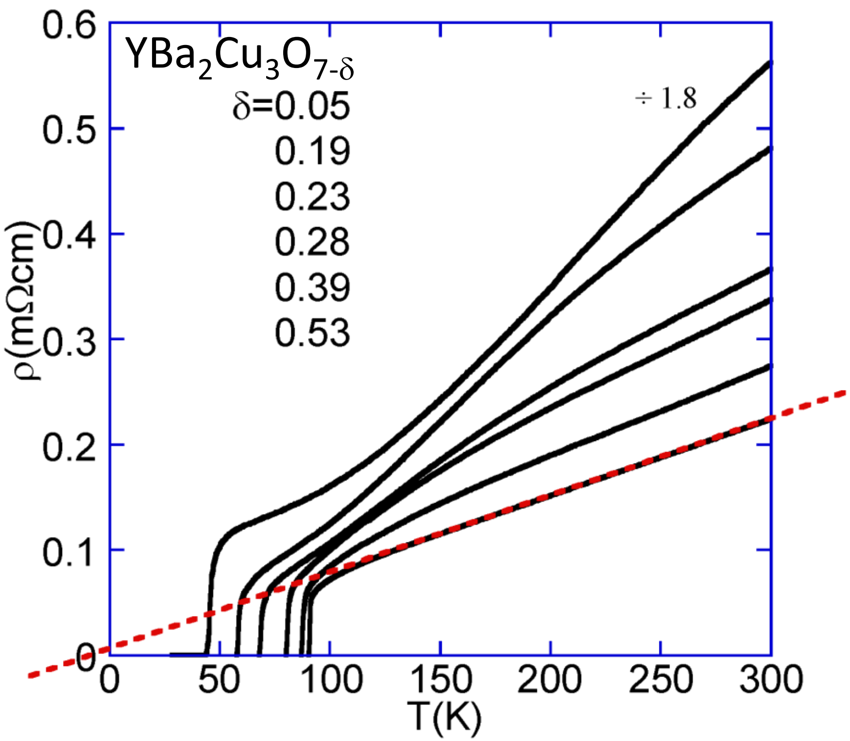
Suggests that mechanism of scattering in cuprates has no energy scale

S. Martin et al, Phys. Rev. B 41, 846 (1990).

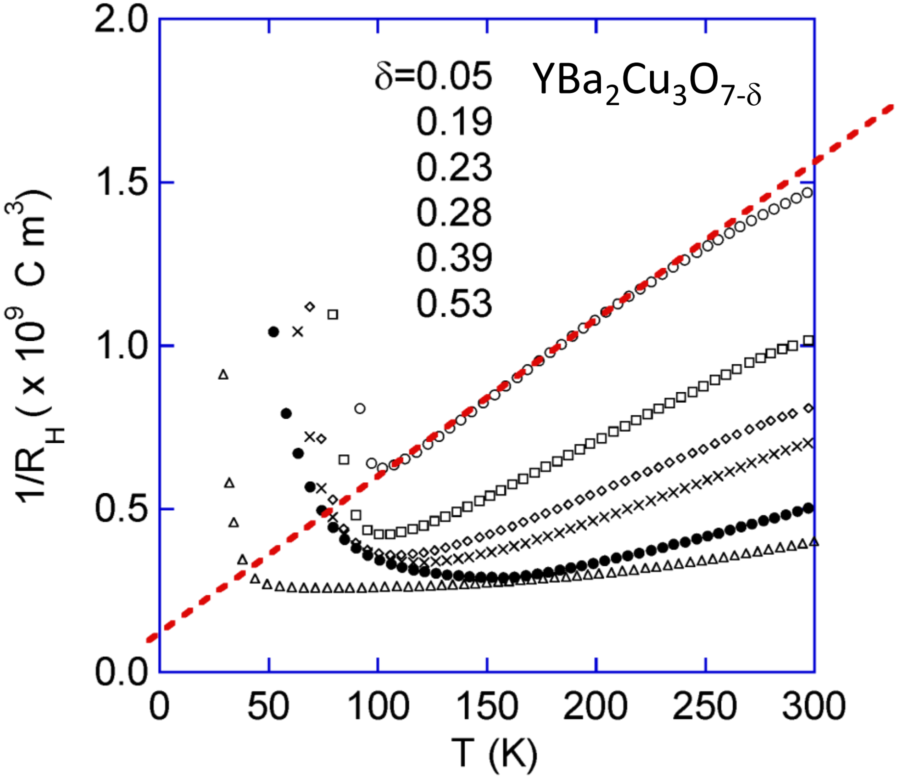
Anomalous normal state : optimal and underdoped

A. Carrington et al. PRB 1993

Resistivity



Hall Coefficient (low field)



Optimal doped

$$\rho \propto T$$

$$1/R_H \propto T$$

Underdoped

Downward curvature in $\rho(T)$

$\rho(T) \rightarrow T^2$ at low T

R_H less T dependent

Simple metals

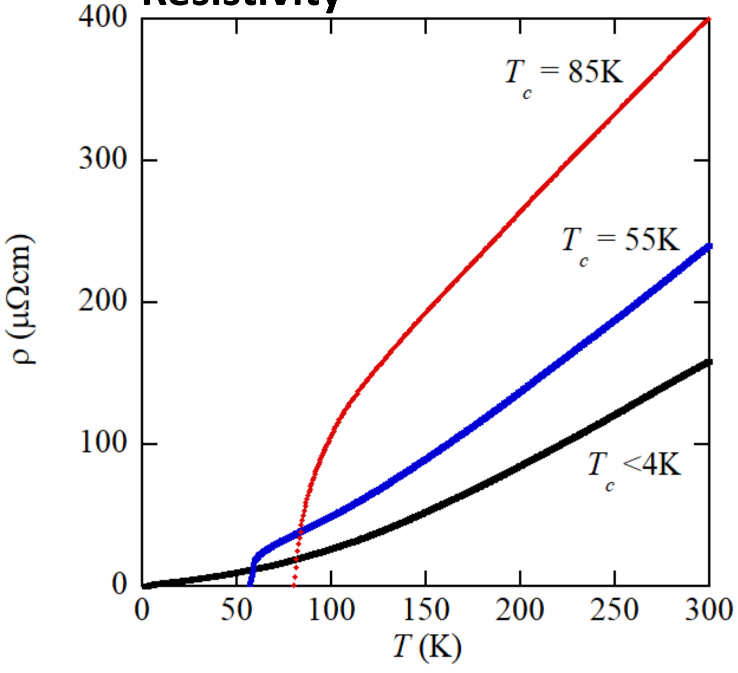
$$\rho \propto T \text{ for } T \gtrsim \theta_D/3$$

$$1/R_H = -ne$$

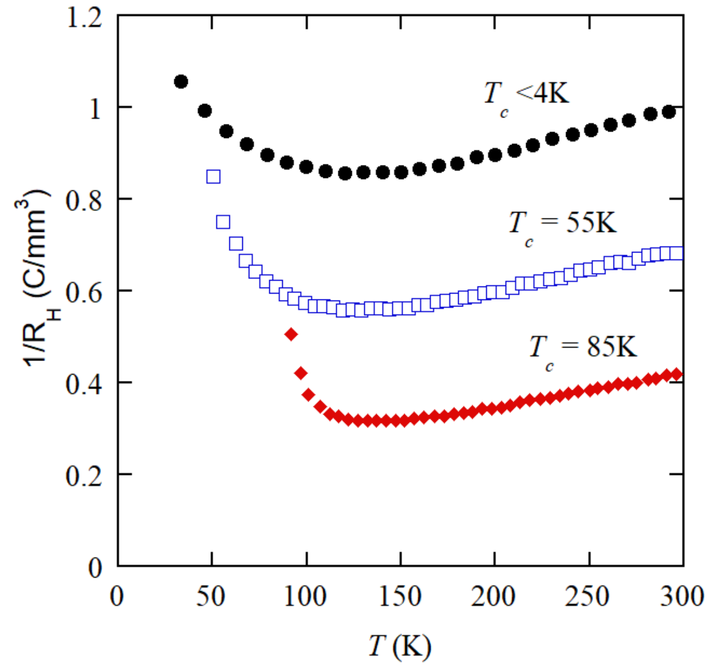
A. Tyler, PhD Thesis, Univ. Cambridge 1997

$Tl_2Ba_2CuO_{6+x}$

Resistivity



Hall Coefficient

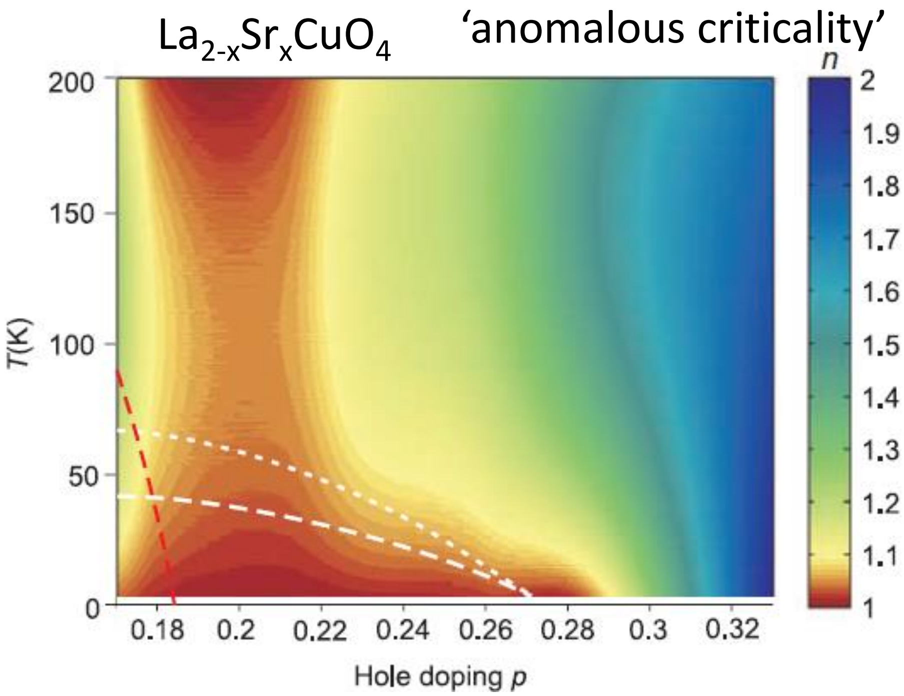


Overdoped

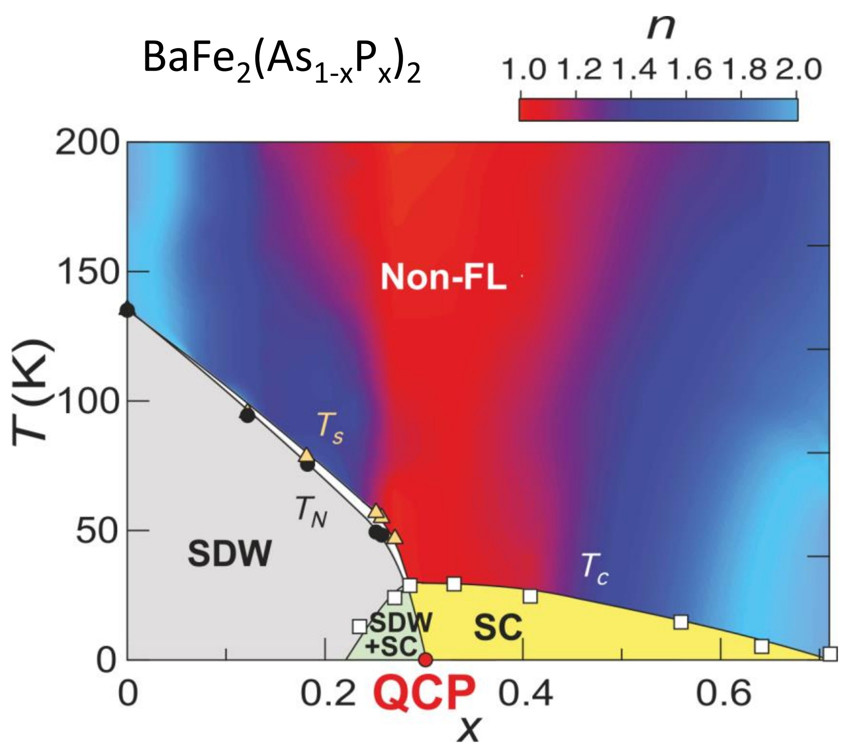
$\rho(T) \rightarrow T^2$

R_H less T dependent

Resistivity exponent



R. Cooper et al Science 2009



S. Kasahara et al., PRB (2010)

$$\rho(T) \sim T^{\alpha}$$

Linear Component in resistivity

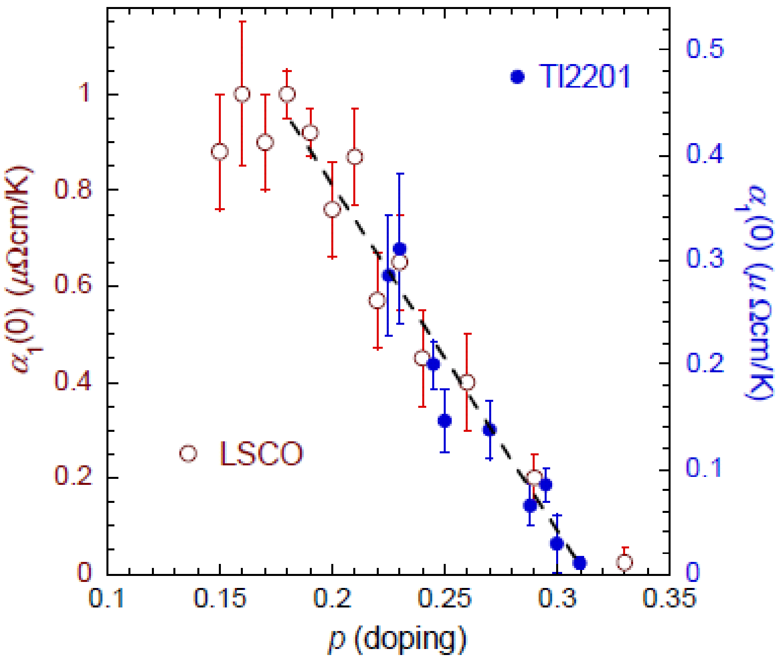
Two ways of viewing the same data

$$\rho(T) = \alpha T + \beta T^2 \qquad \text{or} \qquad \rho(T) \sim T^\alpha$$

Two different current carriers

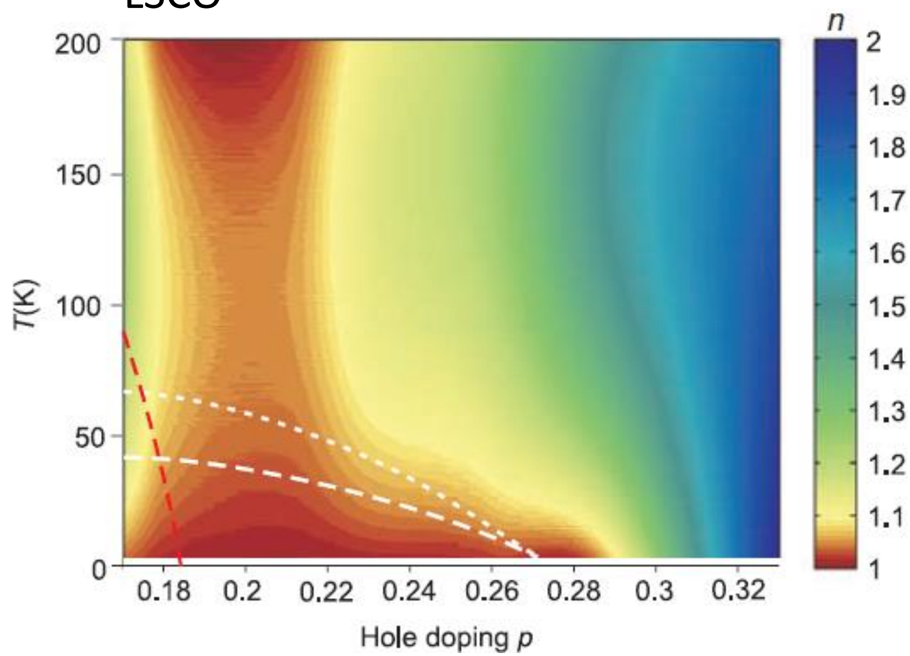
Anisotropic scattering

T linear component



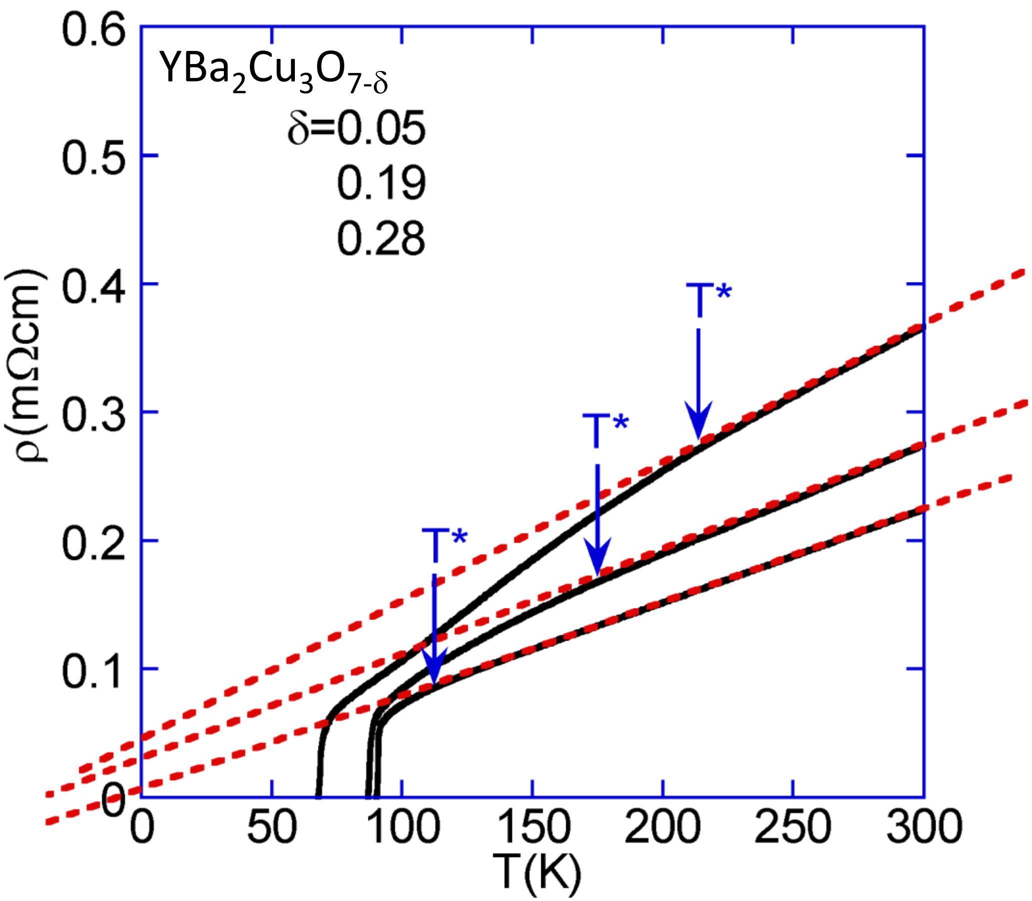
N. Hussey et al Journal of Physics: Conference Series **449** (2013)

LSCO



R. Cooper et al Science 2009

Pseudo-gap in resistivity data

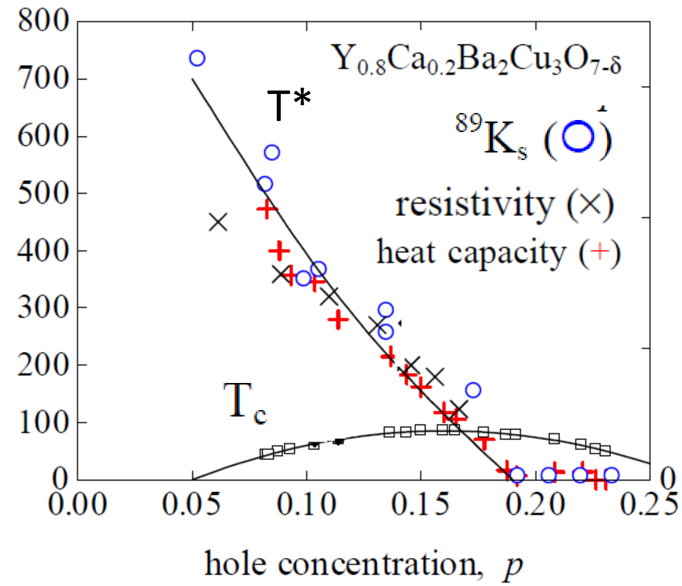


In some other cuprates ρ(T) increases at T*

Is the pseudogap T* a phase transition or energy scale ?
No specific heat anomaly..but abrupt change in torque anisotropy*

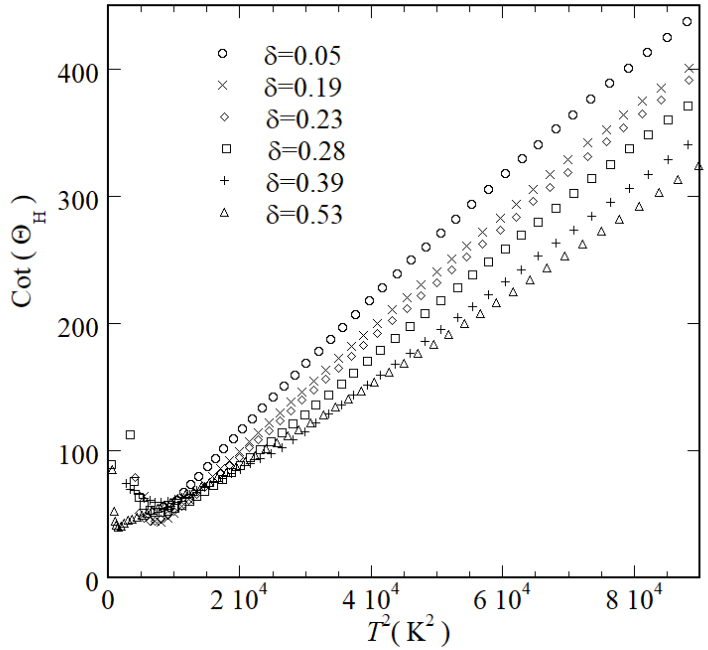
(*Sato et al. Nat.Phys 2017)

J. Loram and J Tallon
Physica C 349, 53 (2001)

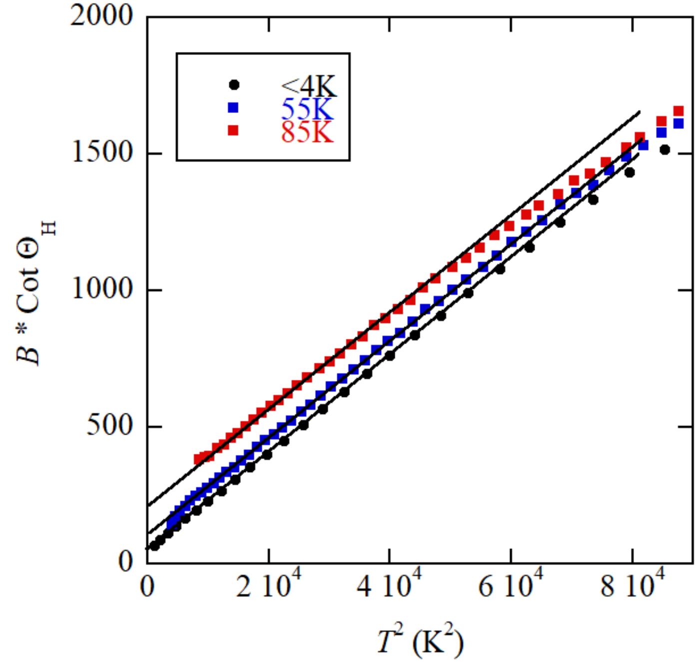


Anomalous normal state : Hall Angle

$YBa_2Cu_3O_{7-\delta}$ →underdoped



$Tl_2Ba_2CuO_{6+x}$ →overdoped



Curvature in ρ(T) cancelled out by same in R_H(T)

$$\cot\theta_H = \frac{\rho_{xx}}{\rho_{xy}} = \frac{\rho_{xx}}{R_H B}$$

in Drude Theory …

$$= \frac{n}{eB}\,\frac{m}{ne^2\tau} = \frac{1}{\omega_c\tau}$$

$\boxed{\cot\theta_H \sim T^2}$ over most of phase diagram

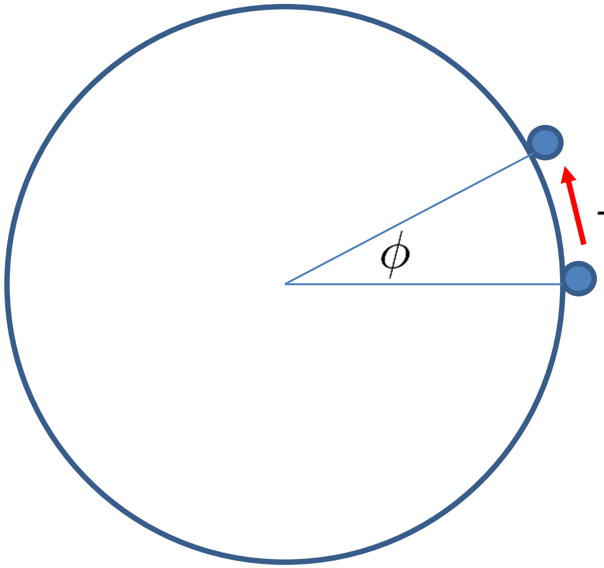
Implies a 'Hall' scattering rate τ_H^{-1}, which is distinct from that for longitudinal scattering from ρ(T)

Chien, Wang,Ong et al PRL 1991

Boltzmann Transport theory : Hall coefficient

Shockley-Chambers tube integral formula (in 2D)

$$\sigma_{xy} = \frac{e^3 B}{2\pi^2 \hbar^2 c \omega_c^2} \int_0^{2\pi} \int_0^{\infty} v_x(\phi) v_y(\phi - \phi') e^{-\frac{\phi'}{\omega_c \tau}} d\phi' d\phi$$



Lorentz force

$$\boldsymbol{F} = e\boldsymbol{v} \times \boldsymbol{B}$$

Boltzmann Transport theory : Hall coefficient (in 2D)

Hall conductivity

$$\sigma_{xy} = \frac{e^3 B}{2\pi^2\hbar^2 c\omega_c^2}\int_0^{2\pi}\int_0^{\infty} v_x(\phi)v_y(\phi-\phi')e^{-\frac{\phi'}{\omega_c\tau}}d\phi' d\phi$$

Longitudinal Conductivity

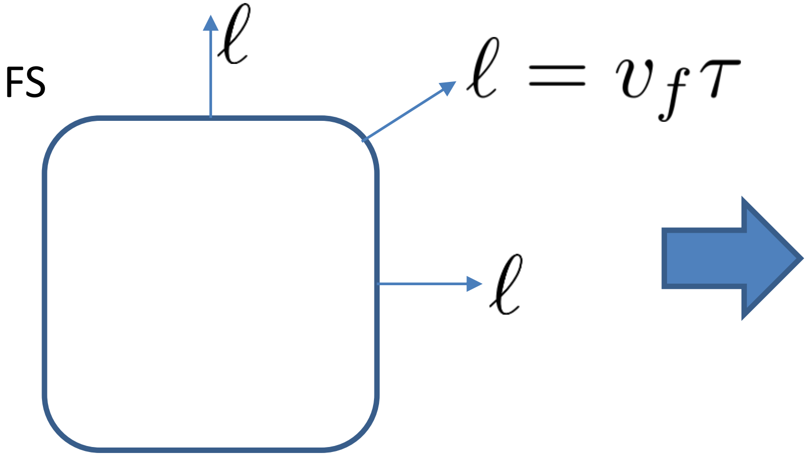
$$\sigma_{xx} = \frac{e^3 B}{2\pi^2\hbar^2 c\omega_c^2}\int_0^{2\pi}\int_0^{\infty} v_x(\phi)v_x(\phi-\phi')e^{-\frac{\phi'}{\omega_c\tau}}d\phi' d\phi$$

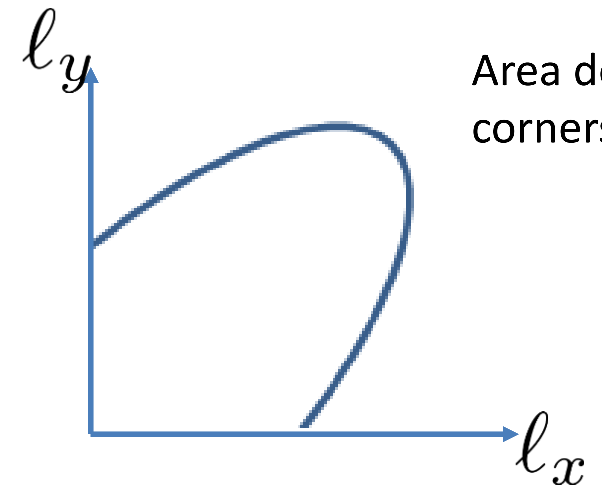
$$R_H B = \rho_{xy} = \frac{\sigma_{xy}}{\sigma_{xx}^2+\sigma_{xy}^2}$$

$$\rho_{xx} = \frac{\sigma_{xx}}{\sigma_{xx}^2+\sigma_{xy}^2}$$

Hall coefficient: weak field

Geometrical construction *(N.P Ong PRB 1991)* Weak field = first order terms only (R_H linear in B)





Area determined by ℓ near corners

Area enclosed by mean-free-path ℓ vector as move around FS determines Hall cond.

$$\boxed{\sigma_{xy} = 2\frac{e^2}{h}\frac{BA_\ell}{\phi_0}}$$

$$\sigma_{xx} = \frac{e^2}{2\pi h} S\ell_{av}$$

$$R_H B = \rho_{xy} = \frac{\sigma_{xy}}{\sigma_{xx}^2 + \sigma_{xy}^2}$$

Spherical Fermi surface, isotropic scattering: $R_H = \dfrac{1}{ne}$

Models of separation of transport lifetimes

$$\tau_\rho^{-1} \sim T \qquad\qquad \tau_H^{-1} \sim T^2$$

Spin-charge separation (Holons and spinons) P.W. Anderson 1991

Quantum-critical metal:
Holographic metal (dissipative and charge-conjugation symmetric currents)
(Blake & Donos, PRL 2015)

Anisotropic scattering (conventional Fermi surface, but unconventional scattering):
Curve parts of FS dominate Hall response
(J.R. Cooper PRL 1992, Stojkovic and Pines PRB 1997)

Weak field : Two bands

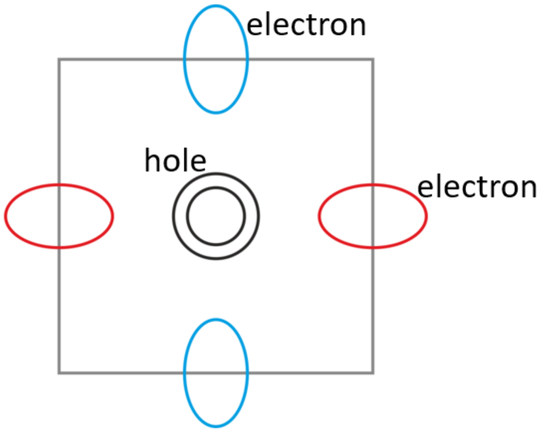
Add the conductivities for each band, assume very weak field (ρ and R_H indep. of H)

$$\sigma_{xy} = \sigma^1_{xy} + \sigma^2_{xy}$$

$$\sigma_{xx} = \sigma^1_{xx} + \sigma^2_{xx}$$

$$\underline{\sigma} = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ -\rho_{xy} & \rho_{xx} \end{pmatrix}^{-1}$$

e.g., iron-based supercond.



$$R_H = \frac{R_1\rho_2^2 + R_2\rho_1^2 + R_1R_2(R_1+R_2)B^2}{(\rho_1+\rho_2)^2 + (R_1+R_2)^2B^2}$$

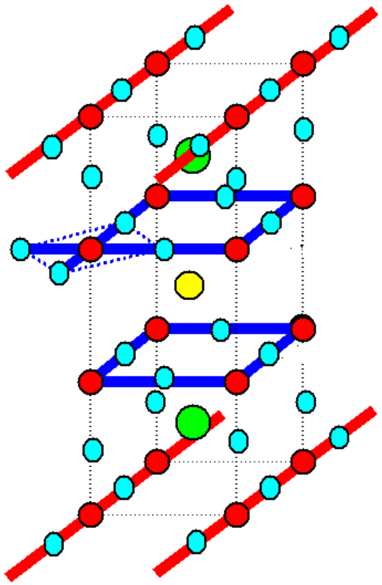
$$\rho = \frac{\rho_1\rho_2(\rho_1+\rho_2) + (\rho_1R_2^2 + \rho_2R_1^2)B^2}{(\rho_1+\rho_2)^2 + (R_1+R_2)^2B^2}$$

Now ρ and R_H have a field dependence even if ρ and R_H in each band do not

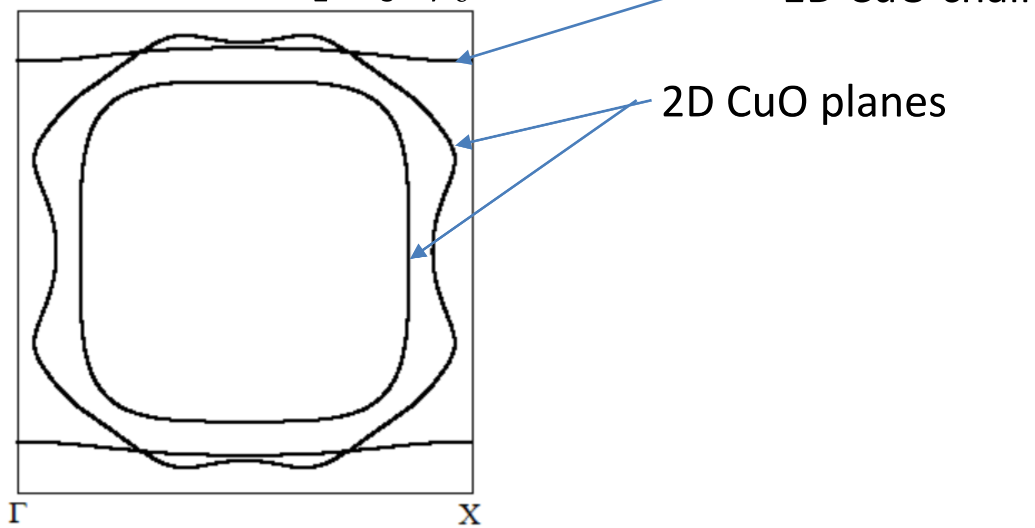
Now a new 'weak field' regime is where B^2 term in R_H is small

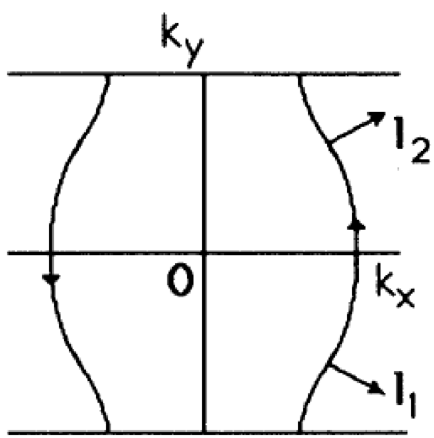
Note this is a different 'weak-field' regime from that where $\omega_c\tau \gg 0$

Two bands in Cuprates: Chains and planes in YBCO

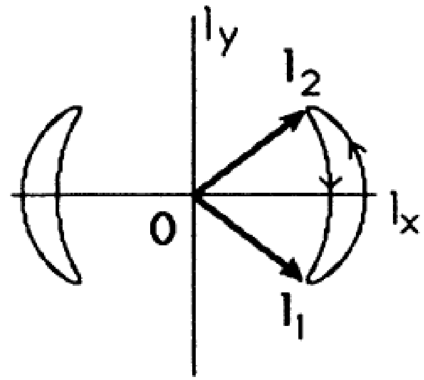


Fermi surface $YBa_2Cu_3O_{7-\delta}$





Fermi Surface

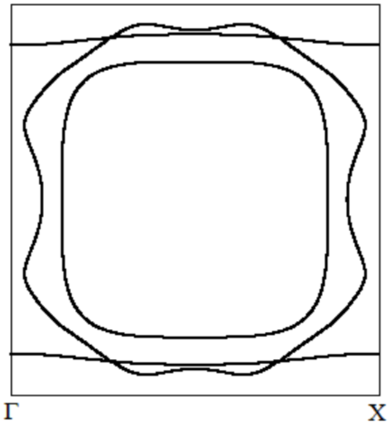


l curve

Quasi 1D Fermi surface

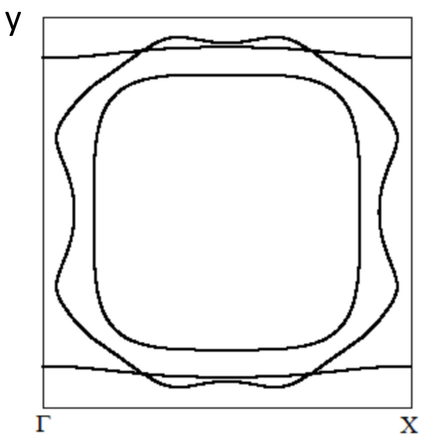
For isotropic ℓ , $\sigma_{xy} = 0$

Chains and planes : add conductivities



$$\sigma_{xy} = \sigma^c_{xy} + \sigma^p_{xy}$$
$$\sigma_{xx} = \sigma^p_{xx} + \sigma^c_{xx}$$
$$\sigma_{yy} = \sigma^p_{yy} + \sigma^c_{yy}$$

Chains and planes



$$\sigma_{xy} = \cancel{\sigma^c_{xy}} + \sigma^p_{xy}$$
$$\sigma_{xx} = \sigma^p_{xx} + \cancel{\sigma^c_{xx}}$$
$$\sigma_{yy} = \sigma^p_{yy} + \sigma^c_{yy}$$

$$R_H B = \rho_{xy} = \frac{\sigma_{xy}}{\sigma_{xx}\sigma_{yy} + \sigma^2_{xy}}$$

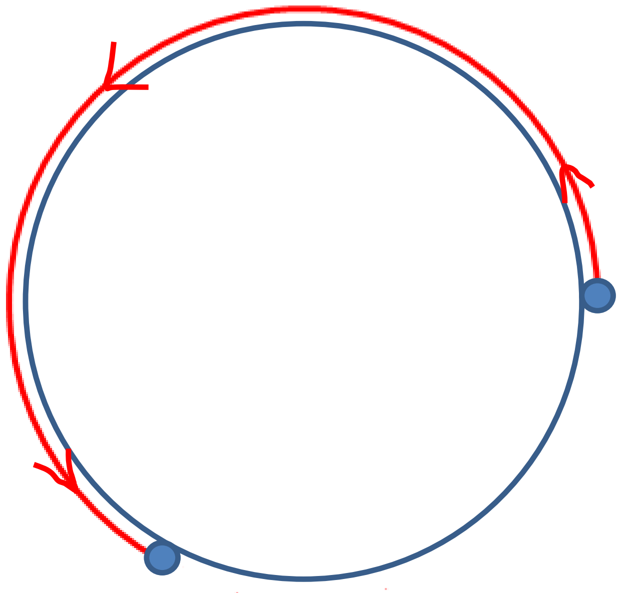
For overdoped YBCO

$$R_H = R^p_H \frac{\rho_b}{\rho_a} \simeq 1.5 R^p_H$$

Hall coefficient : high field

Shockley-Chambers tube integral formula (in 2D)

$$\sigma_{xy} = \frac{e^3 B}{2\pi^2 \hbar^2 c \omega_c^2} \int_0^{2\pi} \int_0^{\infty} v_x(\phi) v_y(\phi - \phi') e^{-\frac{\phi'}{\omega_c \tau}} d\phi' d\phi$$



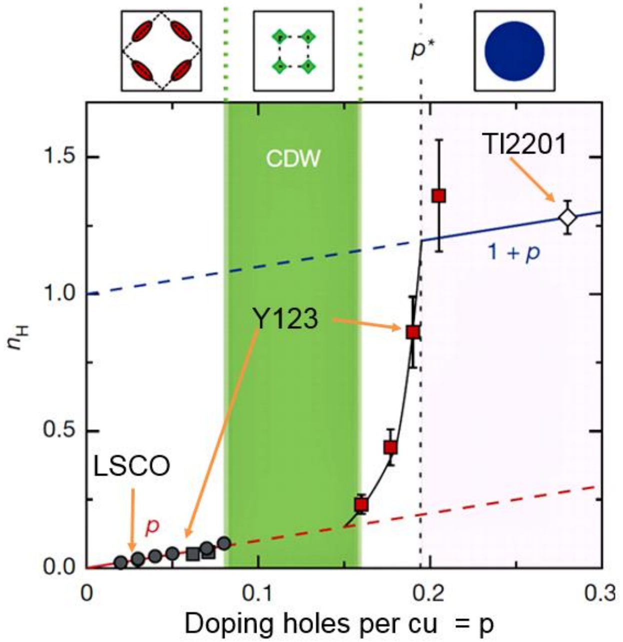
High field limit

$$\omega_c \tau \gg 1$$

$$R_H \to \frac{1}{ne}$$

Recover simple connection between Hall coefficient and Fermi volume in high field limit (single band) for systems with anisotropic scattering or Fermi velocity

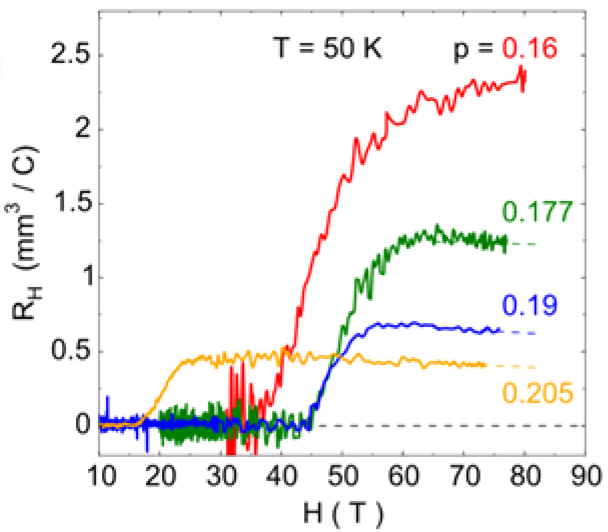
'Abrupt' change in high field, low T, n_H at point where pseudogap starts

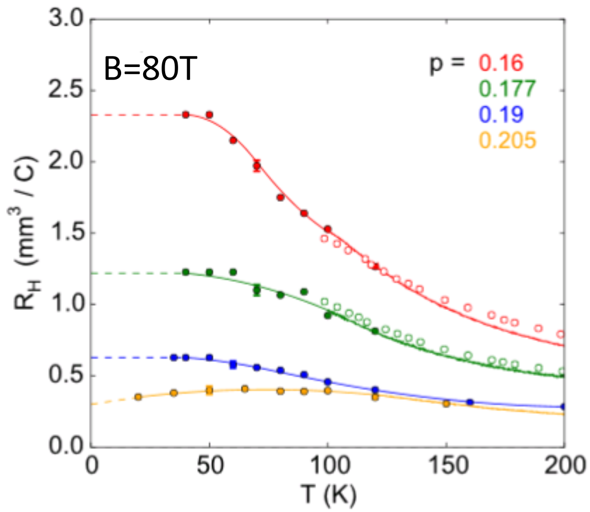


S. Badoux *et al* Nature 2016

Questions

1) How do chains influence n_H
2) Is high field, low T limit reached ?
3) **What happens in far overdoped regime?**



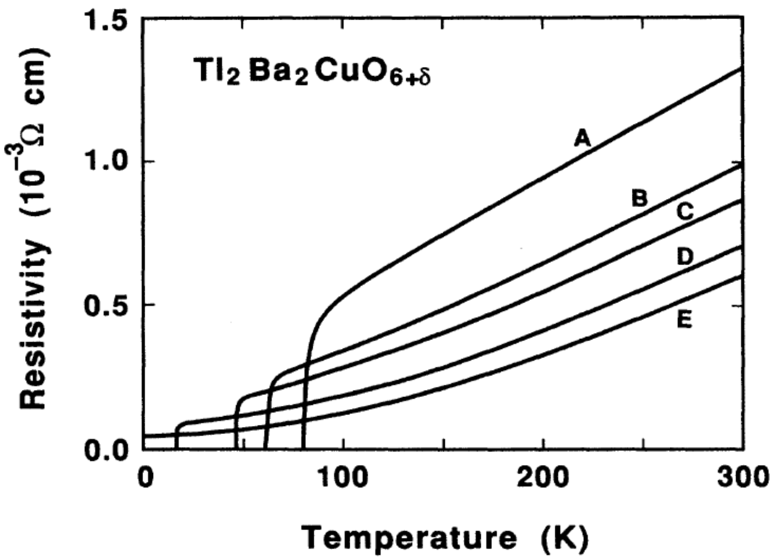


Transport properties in the overdoped regime

$$Tl_2Ba_2CuO_{6+x}$$

$Tl_2Ba_2CuO_{6+x}$: access overdoped regime

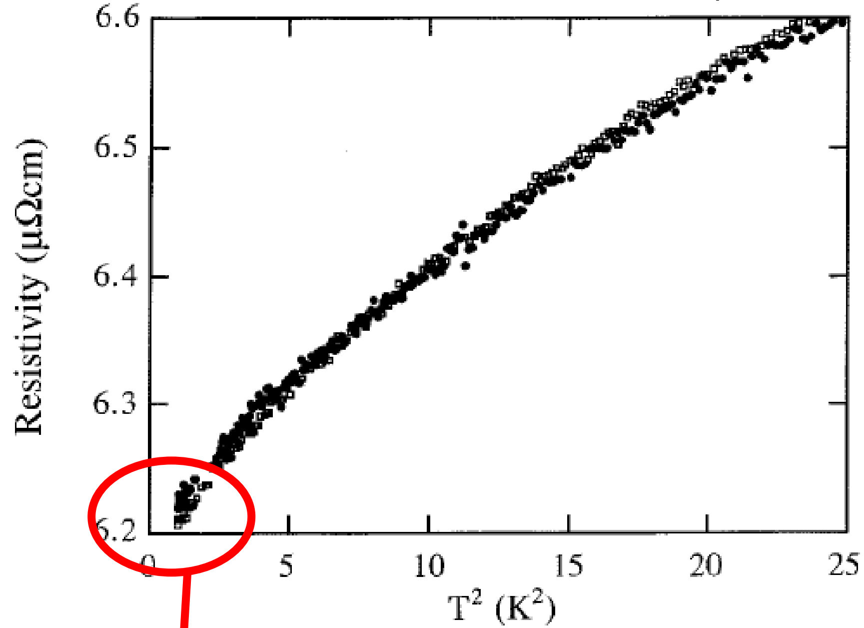
Single Cu layer (no chains)
Electronically clean (cf: LSCO , nd-LSCO)
Single band, no Lifshitz transitions



Kubo et al., PRB 1991

Superficially evolution from ρ~T to ρ~T^2 ?

Tl-2201, T_c = 11K
A.P.Mackenzie et al, PRB 1996

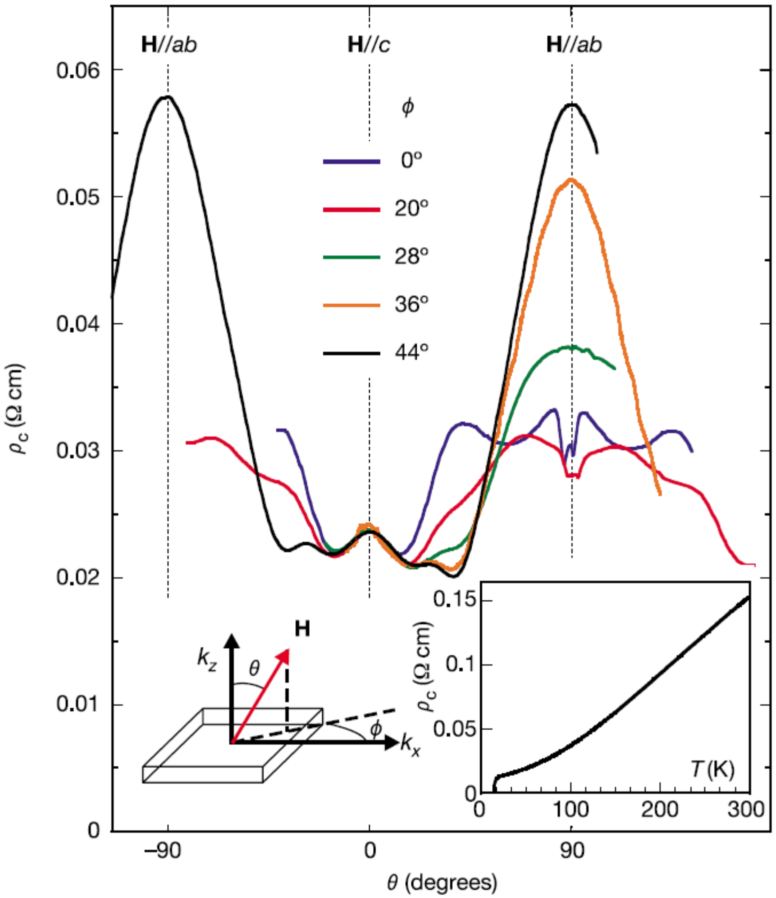


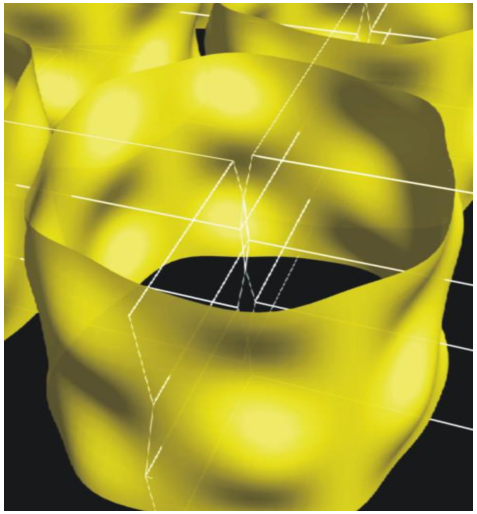
Very low residual resistance, mfp ~ 1000 Å

Far overdoped $Tl_2Ba_2CuO_{6+x}$: what we know

Angular Dependence Magnetoresistance *N.Hussey et al, Nature 2003*

T_c = 20K

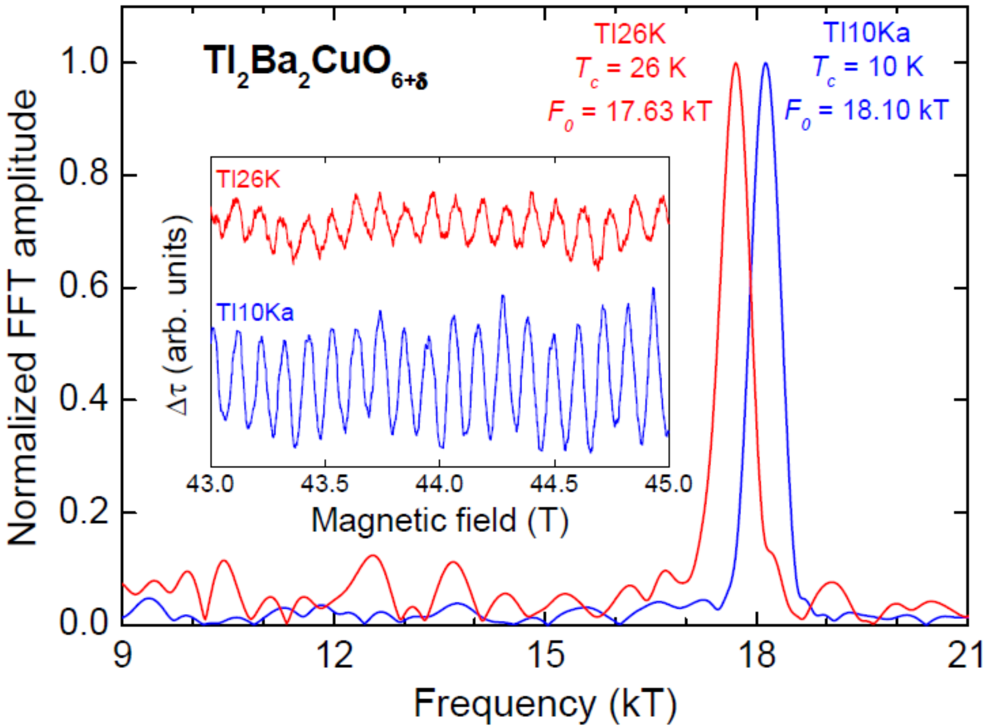




Semiclassically coherent Fermi surface
Highly 2D
Fermi surface shape close to DFT calculations

Far overdoped $Tl_2Ba_2CuO_{6+x}$: what we know

Quantum Oscillations



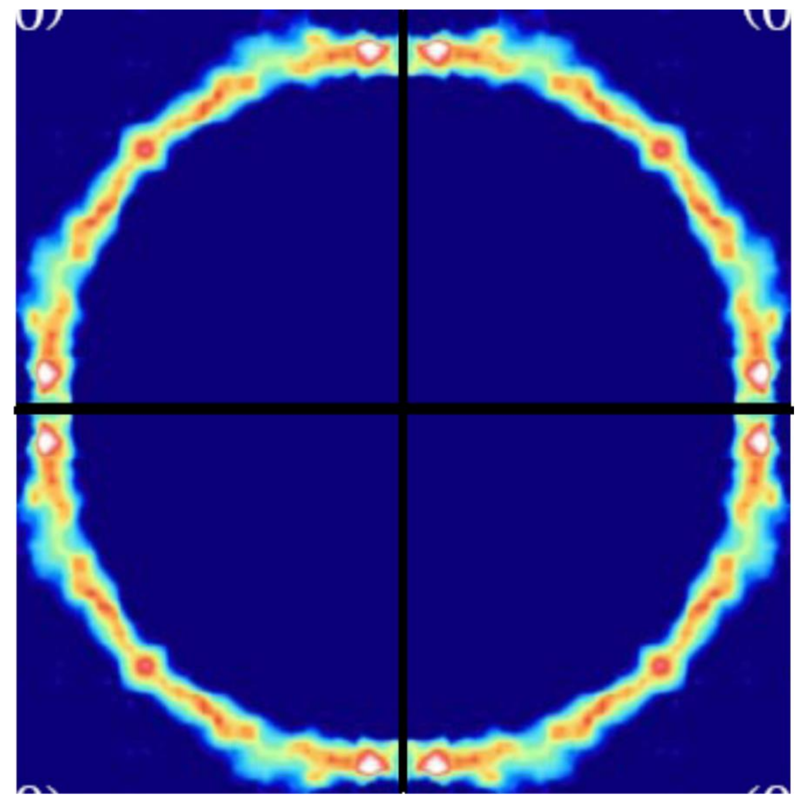
B. Vignolle, N Hussey, AC et al, Nature (2008)

A. Bangura, N Hussey, AC et al, PRB (2010)

P. Rourke, N Hussey, AC et al, NJP (2010)

ARPES

Tc =30K, p = 0.26 ±0.04

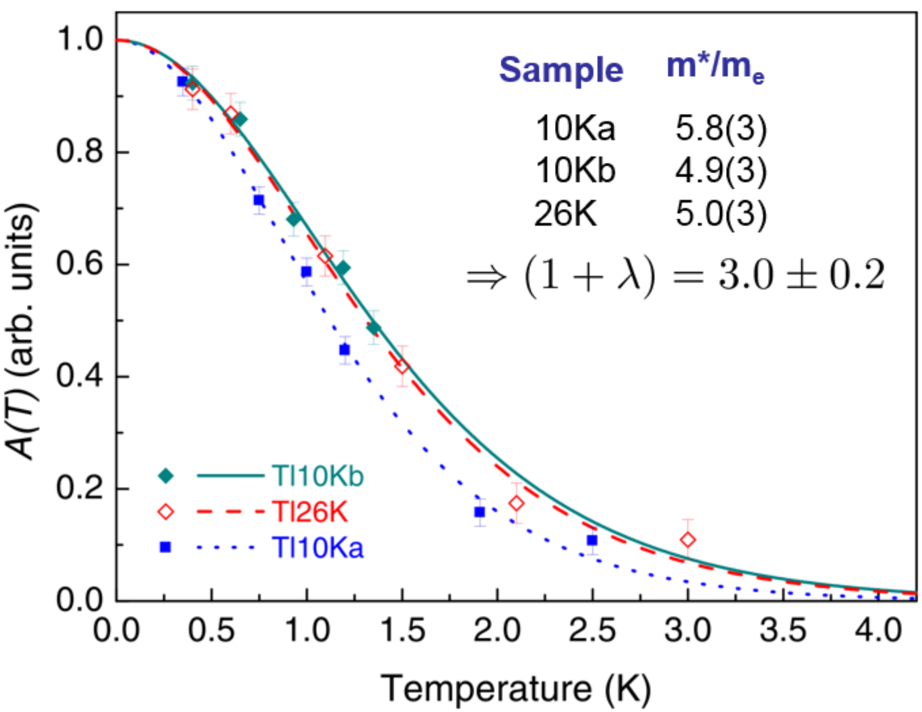


M.Platé et al, PRL (2005)

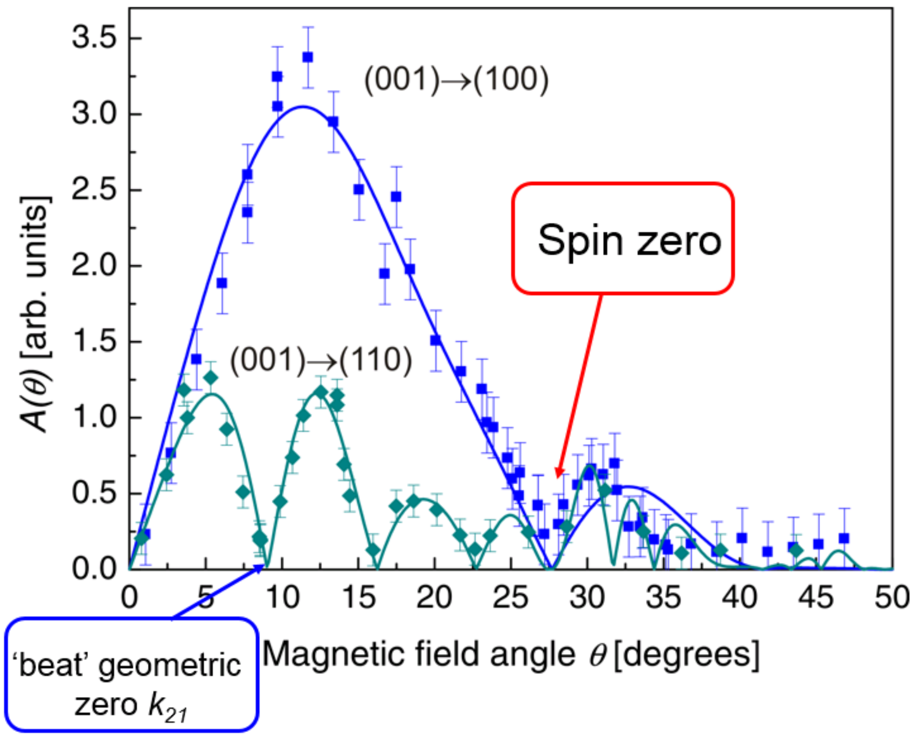
No observation of QO for T_c>26K (even after trying very hard!)

$Tl_2Ba_2CuO_{6+x}$: Fermi surface details

Tl2201: Effective mass determination



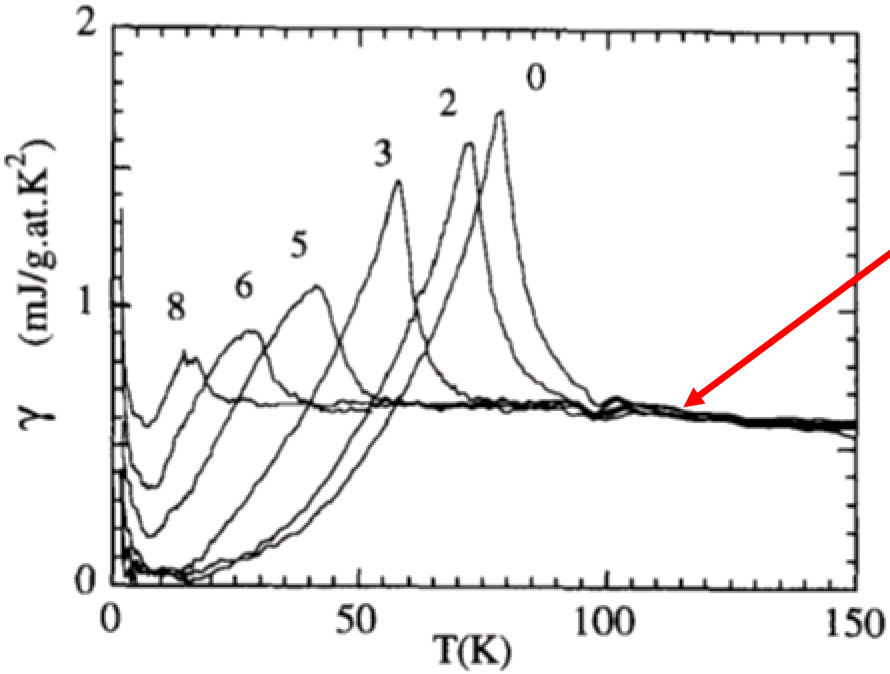
Tl-2201: dHvA Amplitude vs Field angle



Quite strong mass renormalisation, factor 3

Shape of FS determined from field angle dependence of quantum oscillation amplitude, not frequency, because of extreme 2D Fermi surface

Tl2201: Electronic Heat Capacity



John Loram et al., Physica C 1994

γ constant with doping

Specific heat anomalies conserve entropy
So NO pseduogap

Reduction of superconducting anomaly height from pair breaking (disorder) when Tc is low

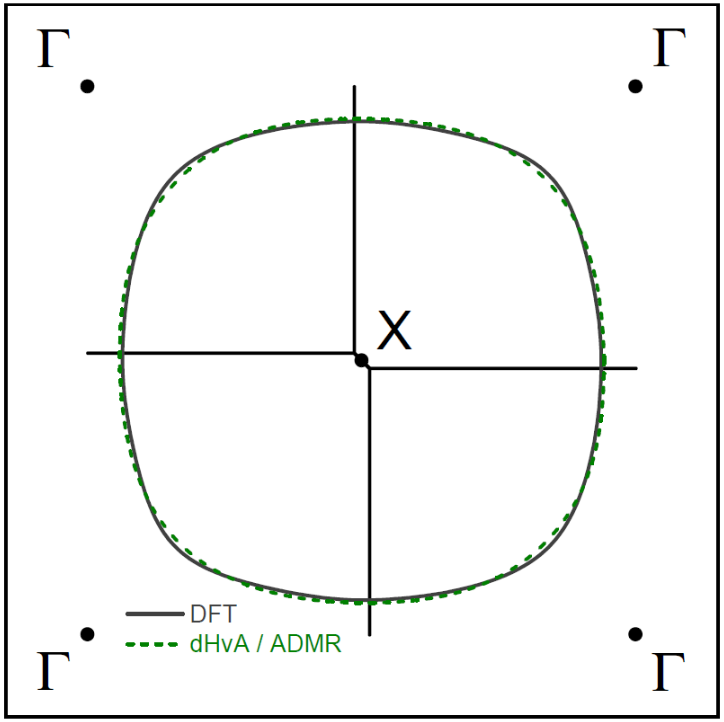
$$\gamma(T_c^+) = 7 \pm 1\mathrm{mJ/molK^2}$$

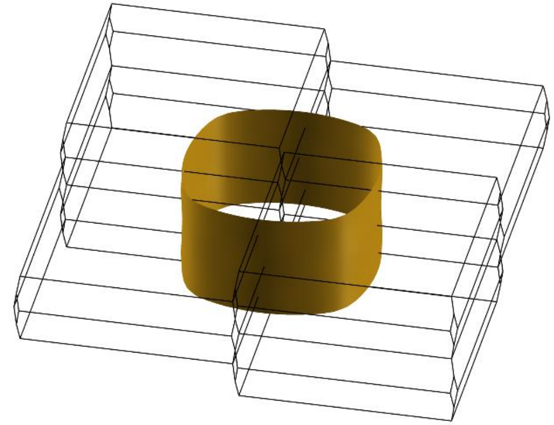
Cf: dHvA mass $m^* = 5.2 \pm 0.4 m_e$

$$\gamma_{\mathrm{dHvA}} = 7.6 \pm 0.6\mathrm{mJ/mol/K^2}$$

Measure electronic specific heat in good agreement with QO mass
So….
No missing FS sheets
Mass field independent

Compare measured Fermi surface to DFT





(a) $k_{0,0}$ (b) $k_{0,0} + k_{4,0}$ (c) $k_{0,0} + k_{2,1}$ (d) $k_{0,0} + k_{6,1}$ (e) $k_{0,0} + k_{10,1}$

	$k_{0,0}$ (Å^{-1})	$\frac{k_{4,0}}{k_{0,0}}$	$\frac{k_{8,0}}{k_{0,0}}$	$\frac{k_{12,0}}{k_{0,0}}$	$\frac{k_{16,0}}{k_{0,0}}$	$k_{2,1}$ (Å^{-1})	$\frac{k_{6,1}}{k_{2,1}}$	$\frac{k_{10,1}}{k_{2,1}}$
DFT	0.7390	-0.047	0.0088	-0.00135	0.000436	-0.00287	0.50	-0.34
Exp	0.7416	-0.032	—	—	—	-0.00170	0.71	-0.25
	dHvA	ADMR				dHvA	ADMR	ADMR

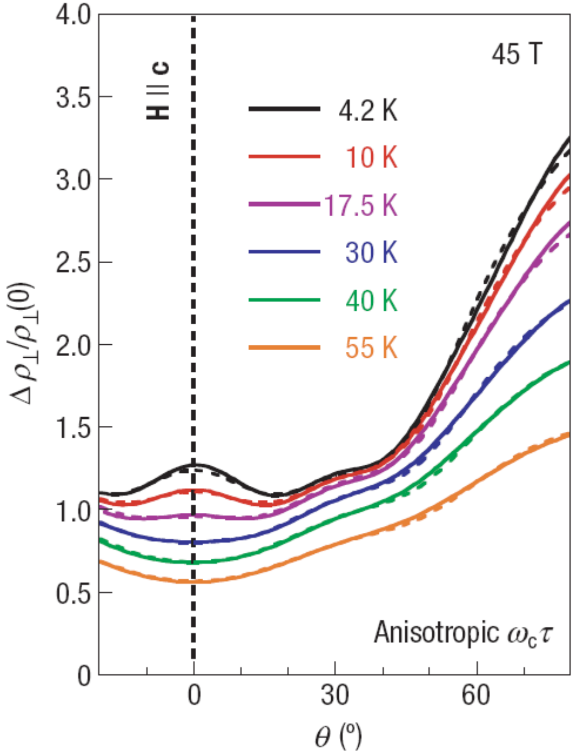
$$\frac{k_{0,0}}{k_{2,1}} = 436$$

Details of Fermi surface warping agree well with DFT

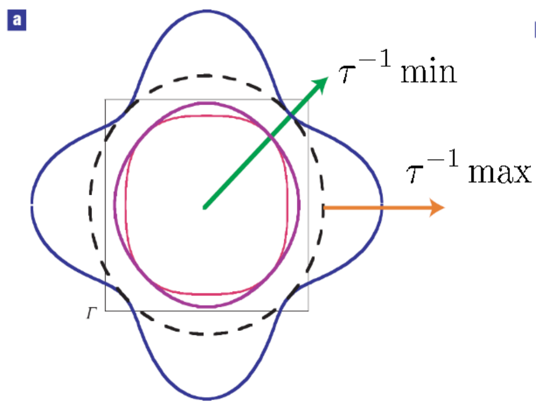
Tl2201: AMDR: Scattering rate

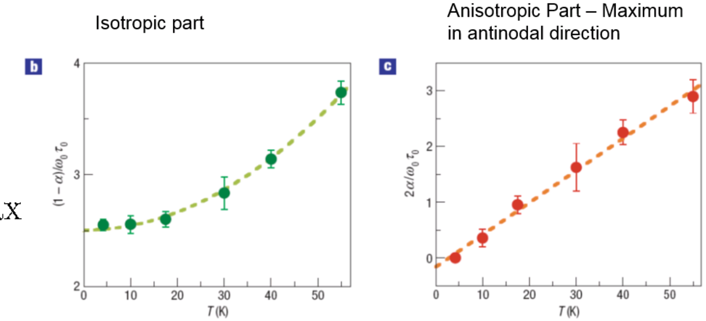
Temperature dependence of ADMR

M. Abdel-Jawad et al, Nature Physics 2006



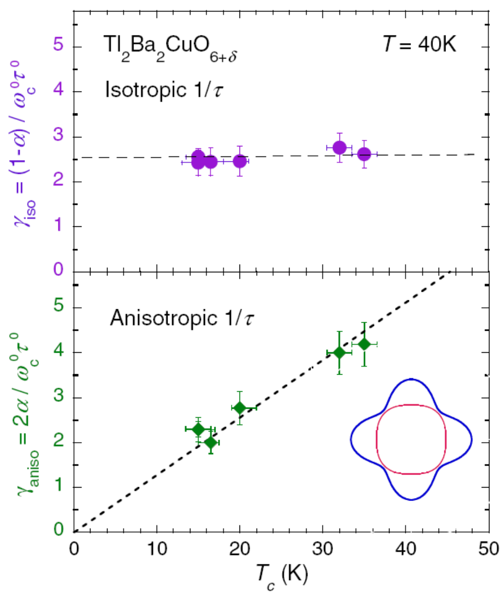
Scattering rate is anisotropic





$\tau^{-1}A + B \sim T^2$ $\qquad$ $\tau^{-1} \sim T$

M. Abdel-Jawad et al, PRL 2007



Isotropic T^2 term does not change with T_c

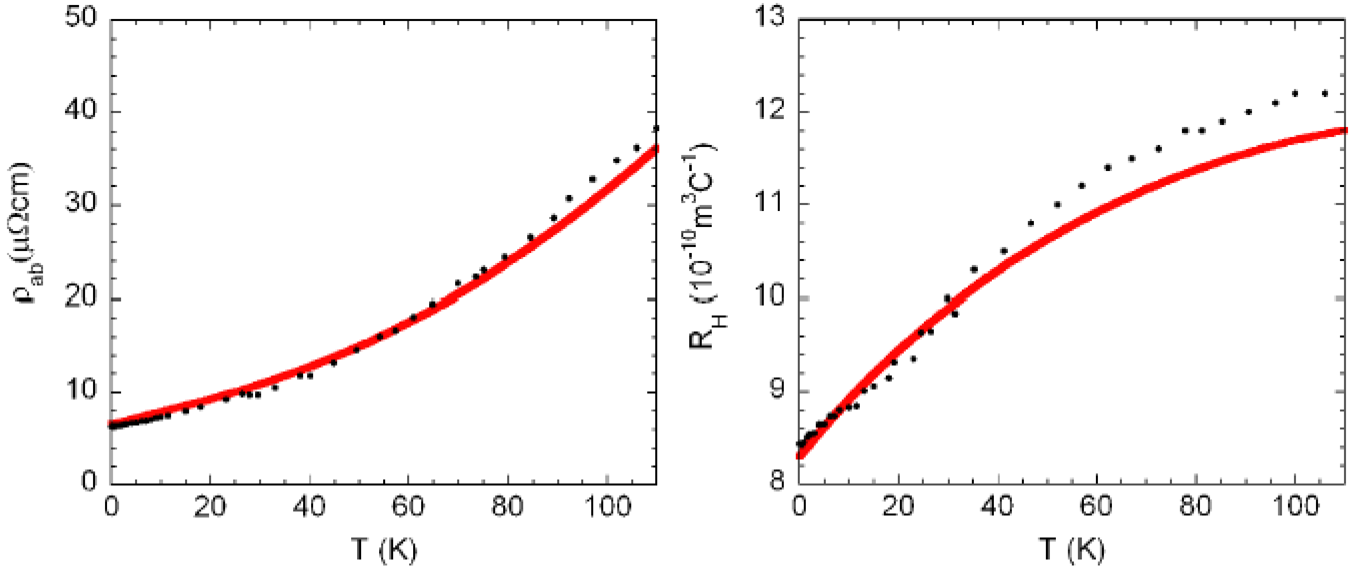
Anisotropic T term scales with T_c

But cannot fit data for T_c>45K with same model

Low field Hall effect+ simulation: Tl2201

M. Abdel-Jawad et al, Nature Physics 2006

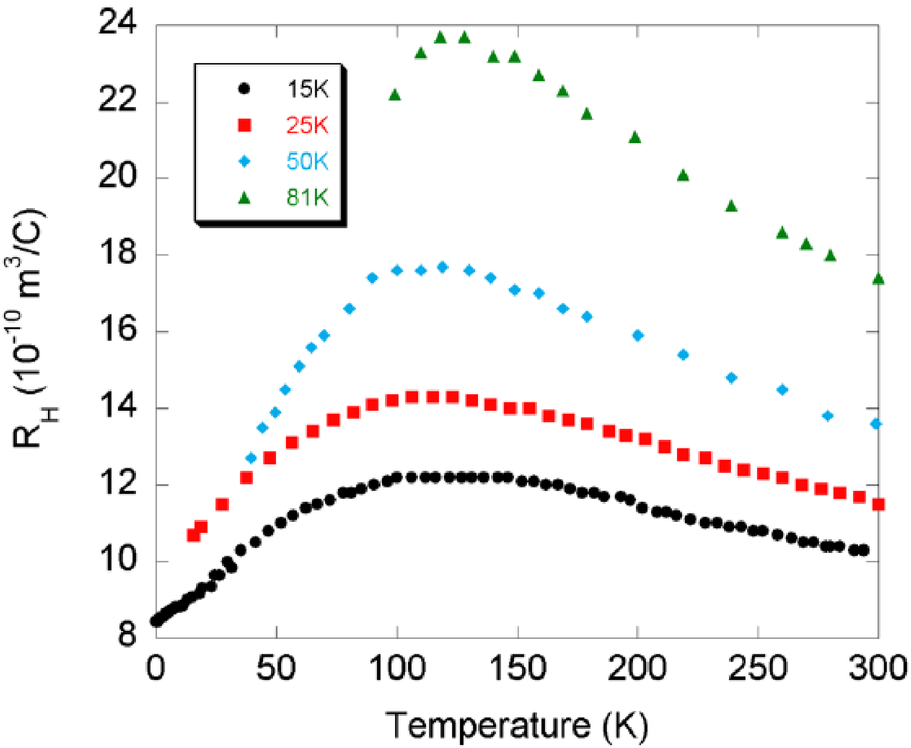
Low field limit, Tl-2201 parameter and data : Tc=15 K

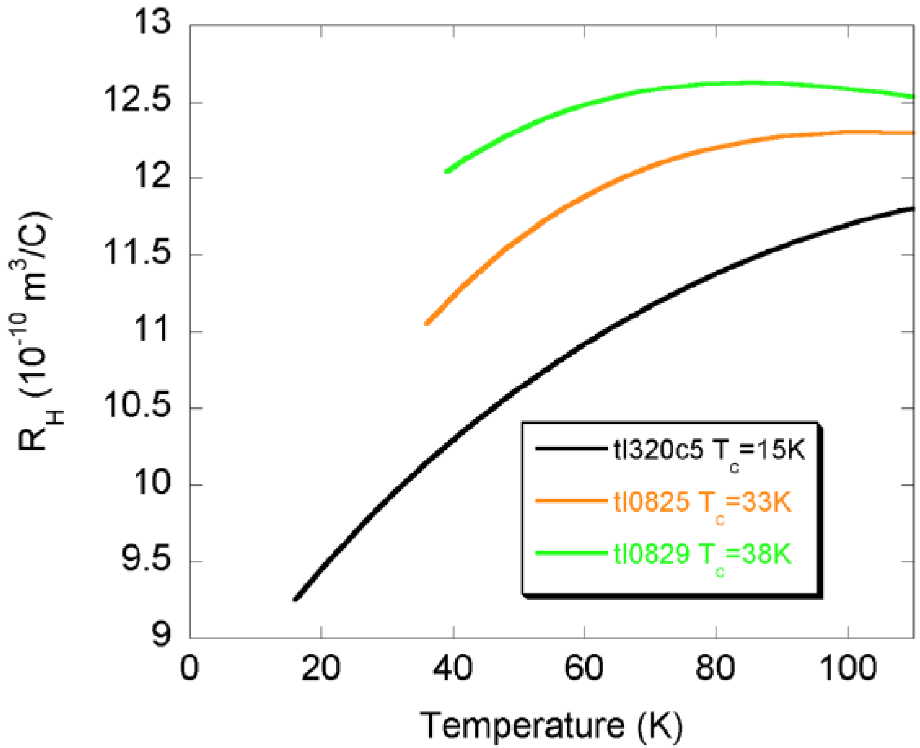


Works well for most overdoped samples

Low field Hall effect+ simulation: Tl2201

M. French : PhD Thesis 2009



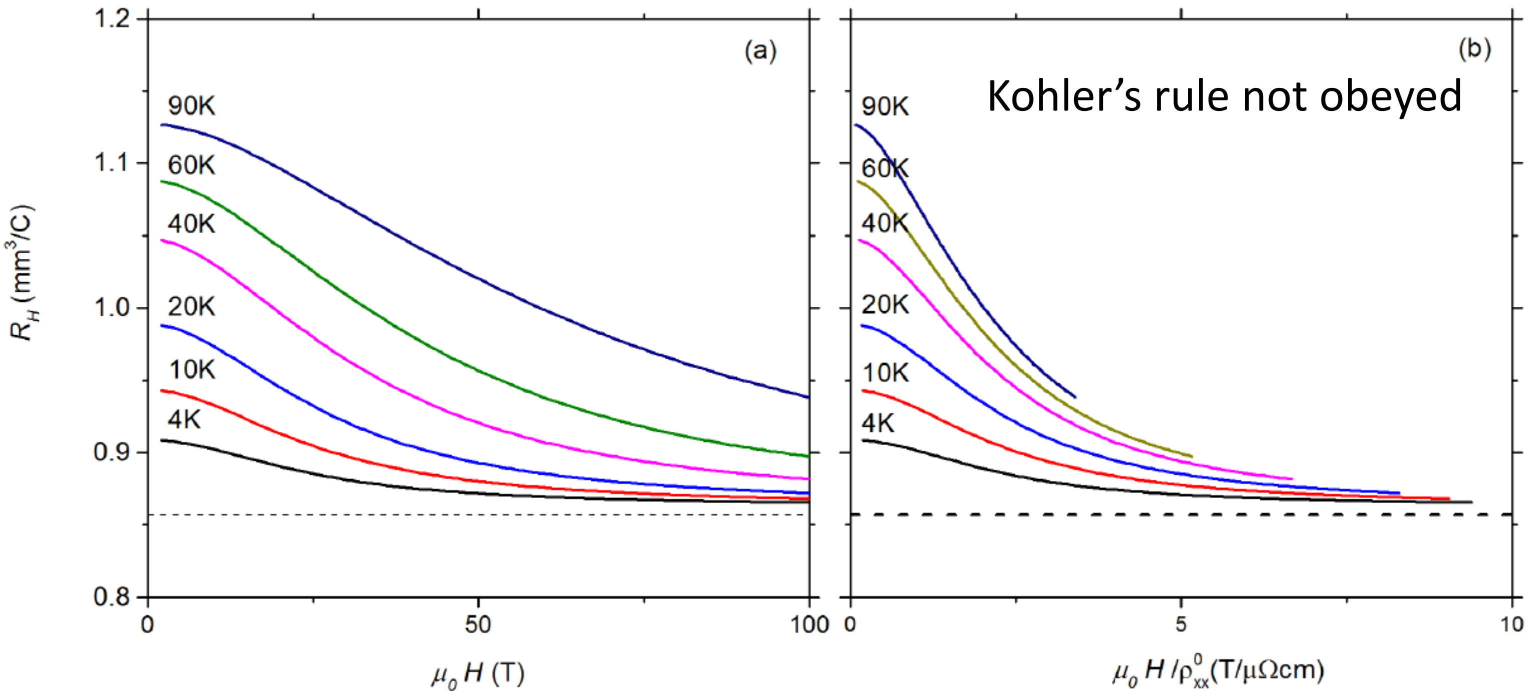


Experimental T dependence is far stronger than theory for higher T_c samples

High field R_H for Tl-2201 : Calculatio

Shockley-Chambers formula to simulate within Boltzmann theory at arbitrary H
Parameters: measured anisotropy of scattering rate + Fermi surface geometry for Tl-2201

Parameters for Tl-2201 : T_c=20K



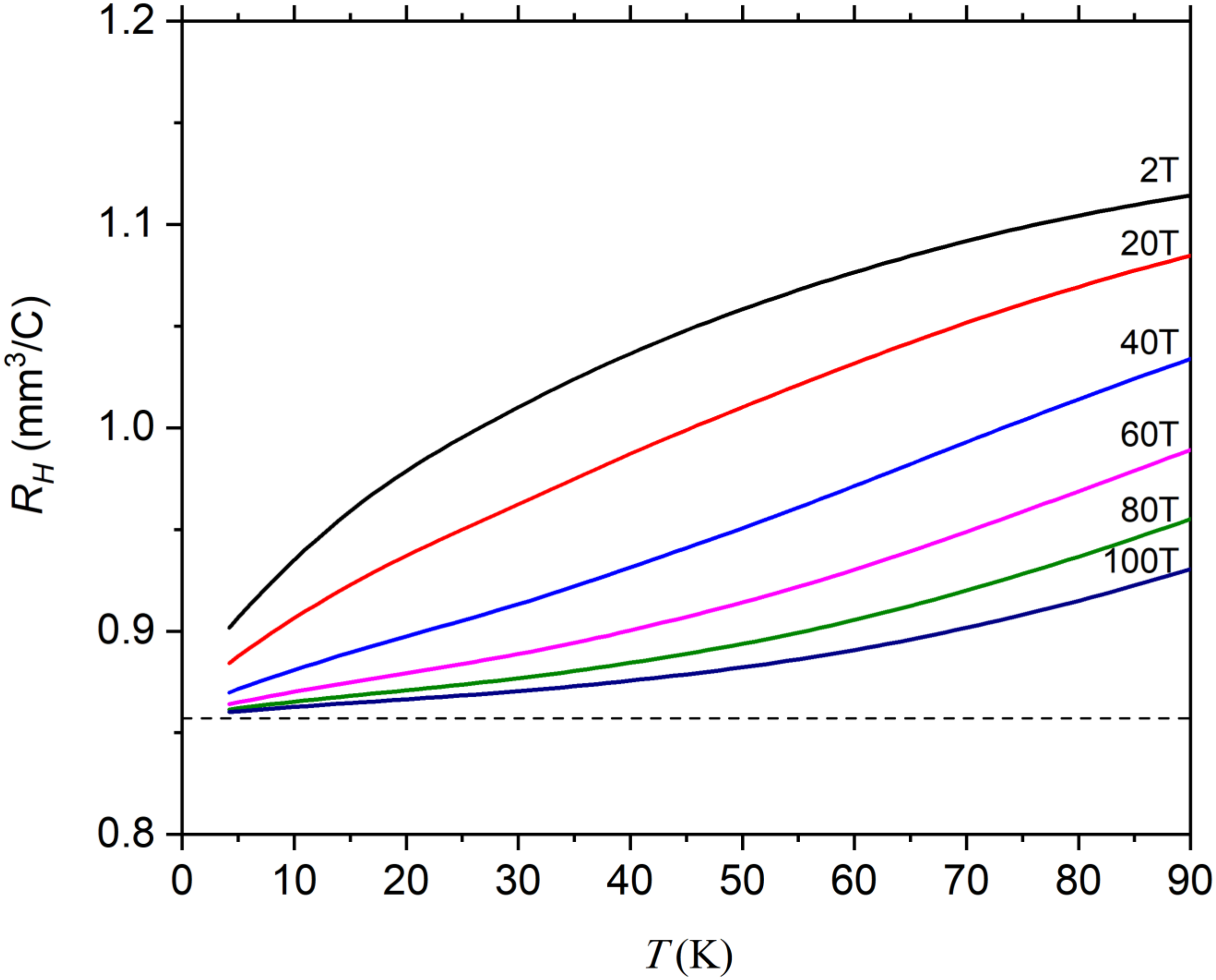
$$R_H = 1/ne$$

R_H tends to 1/ne at low T (isotropic scattering) or high B ($\omega_c\tau$ >>1)

C. Putzke et al arXiv:1909.08102

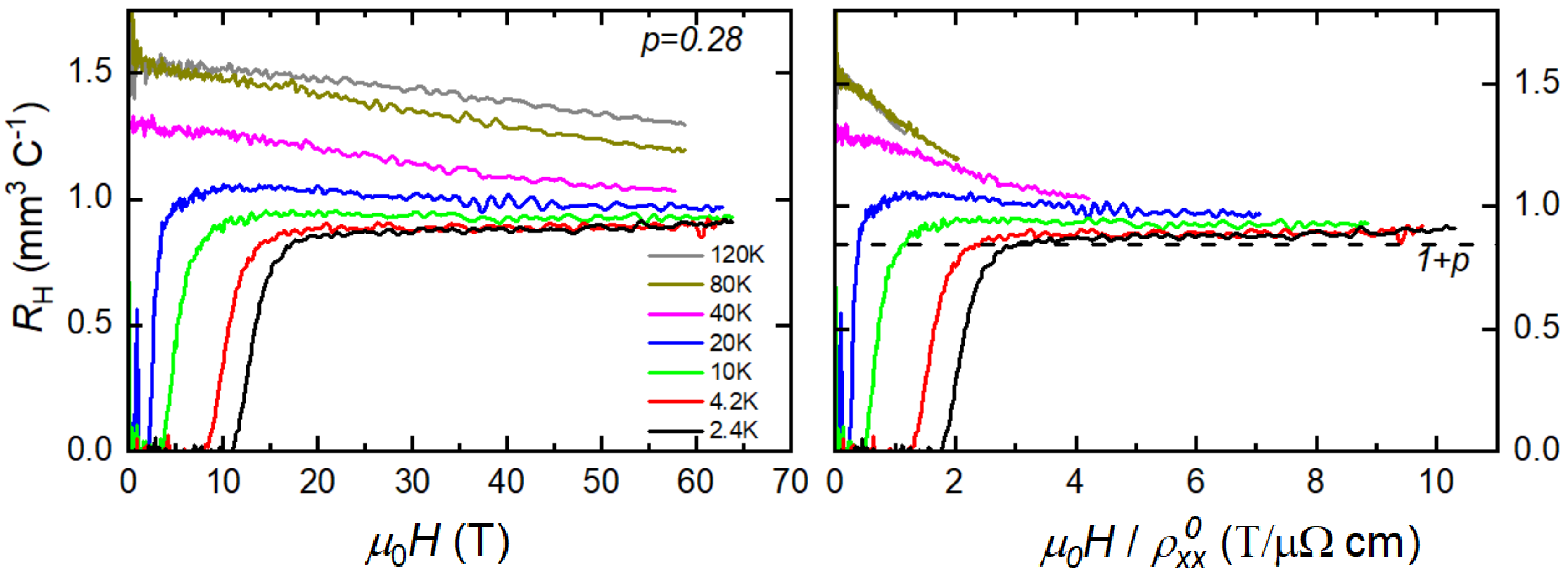
Simulate field dependence of R_H for Tl-2201

T dependence at fixed field



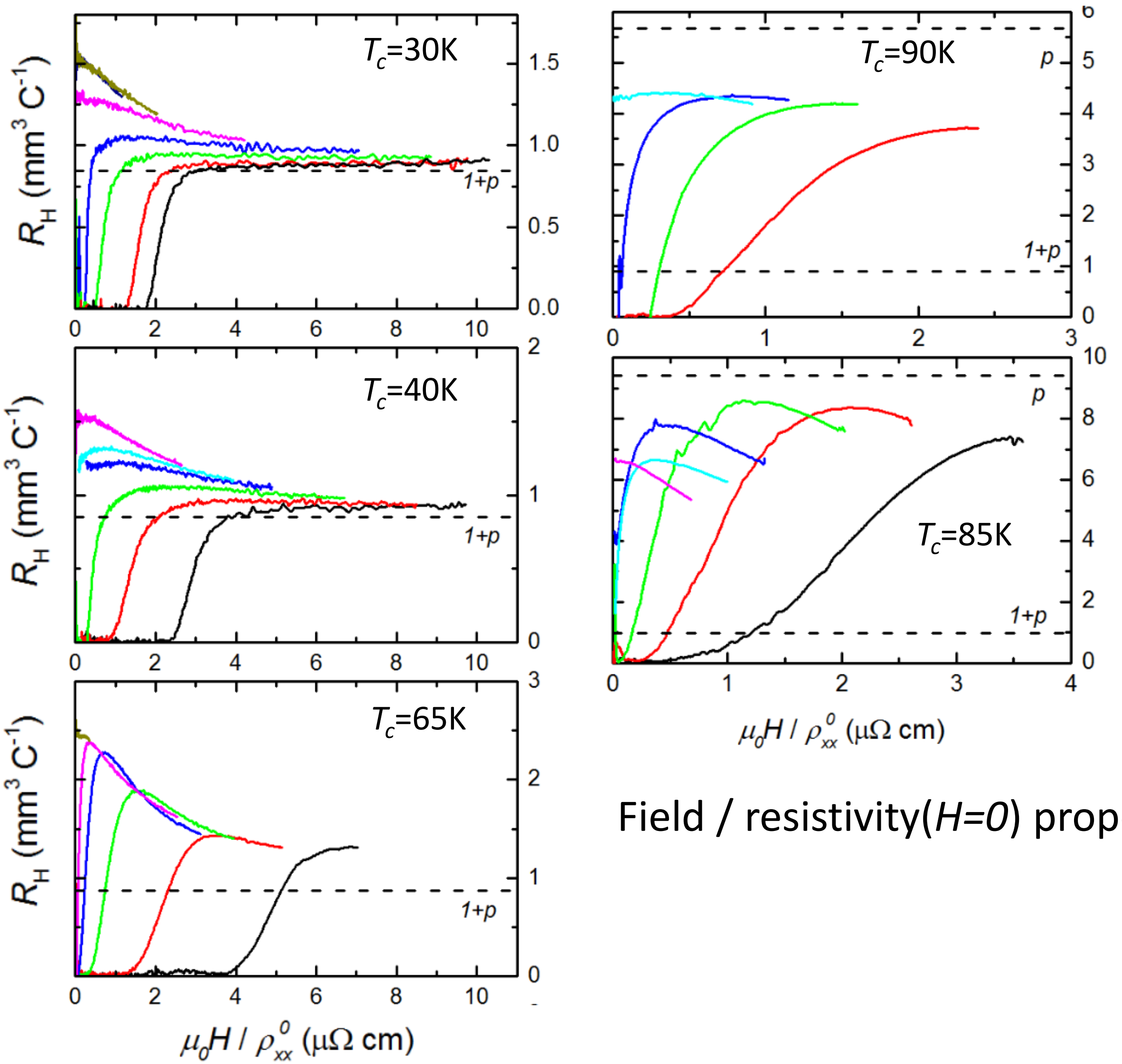
High field suppresses T dependence

High field Hall coefficient in Tl2201



C. Putzke et al arXiv:1909.08102

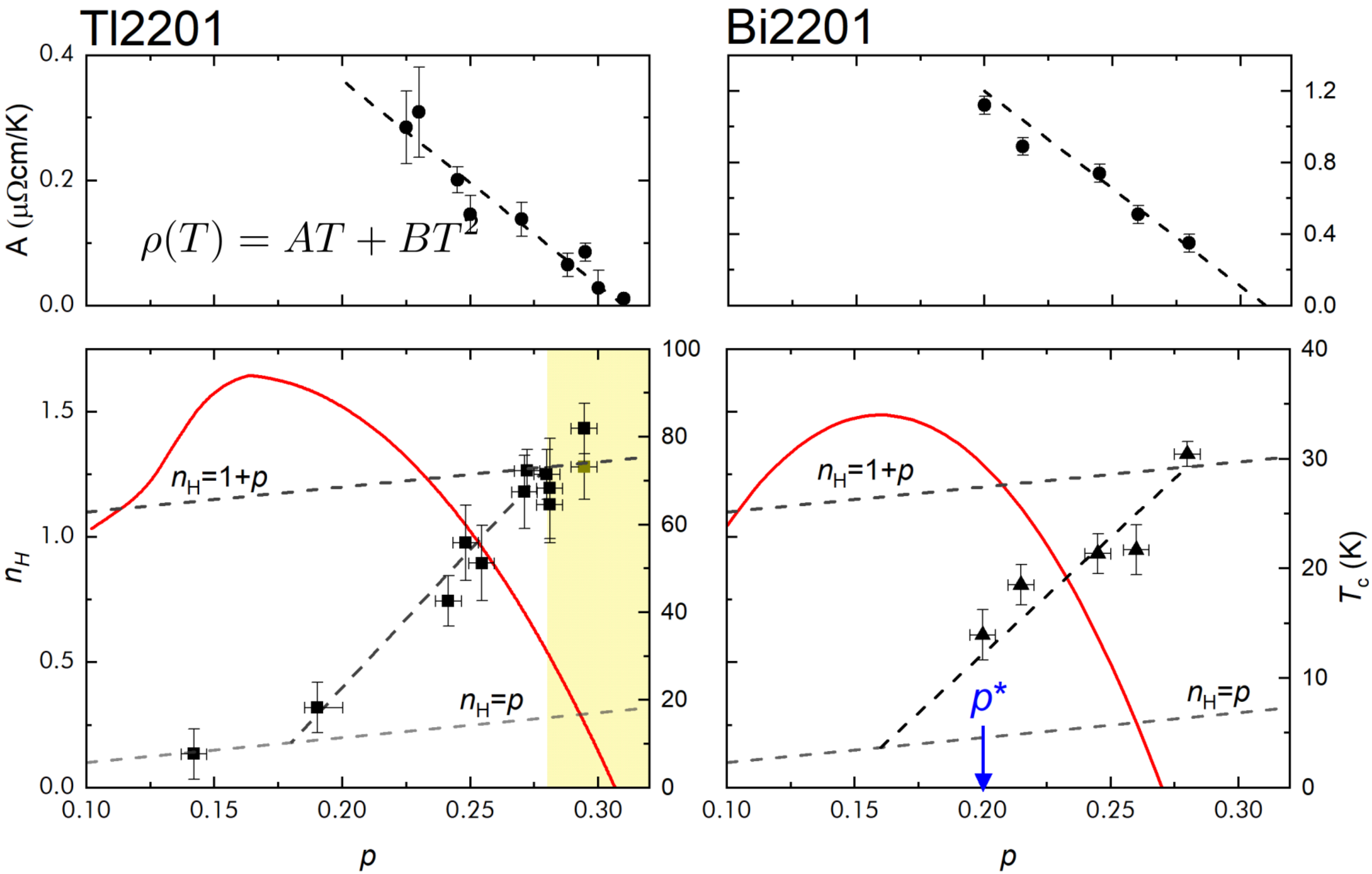
Hall data with scaled x-axis



Field / resistivity($H=0$) proportional to $\omega_c\tau$

C. Putzke et al arXiv:1909.08102

Evolution of n_H in overdoped regime



C. Putzke et al arXiv:1909.08102

Interpretation

Pseudogap hidden below T_c with critical point in far OD regime

❌ no rise of R_H below T_c (H=0)

Anisotropic scattering at T=0

❌ High field should quench this effect

Density wave, causing reconstruction of FS, for T_c>40K in overdoped regime

No evidence for this.

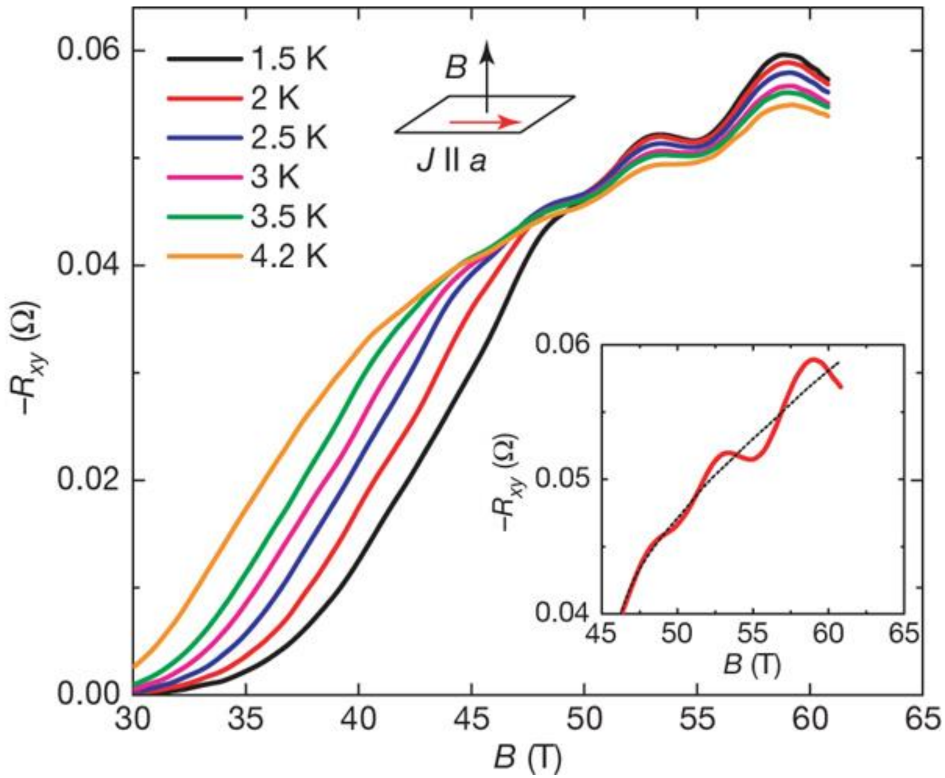
Non-conventional (Boltzmann) transport
incoherent or non-Fermi liquid transport

Potentially could also explain evolution of
linear-in-T term in $\rho(T)$ with doping.

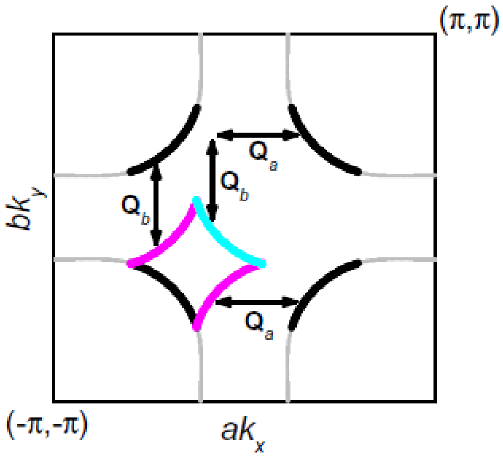
Quantum oscillations in underdoped cuprates

N. Doiron-Leyraud et al., Nature 2007

$YBa_2Cu_3O_{6.52}$



CDW Fermi surface reconstruction



- Impossible from Fermi arcs.
- Inconsistent with band-structure (observed Fermi surface is only 2% of the Brillouin zone area)
- Must be some form of Fermi surface reconstruction due to change of Brillouin zone

Conductivity tensor

$$\mathbf{J} = \sigma \cdot \mathbf{E} \qquad \sigma = \begin{pmatrix} \sigma_{\mathrm{xx}} & \sigma_{\mathrm{xy}} & \sigma_{\mathrm{xz}} \\ \sigma_{\mathrm{yx}} & \sigma_{\mathrm{yy}} & \sigma_{\mathrm{yz}} \\ \sigma_{\mathrm{zx}} & \sigma_{\mathrm{zy}} & \sigma_{\mathrm{zz}} \end{pmatrix}$$

$$\rho = \begin{pmatrix} \rho_{\mathrm{xx}} & \rho_{\mathrm{xy}} & \rho_{\mathrm{xz}} \\ \rho_{\mathrm{yx}} & \rho_{\mathrm{yy}} & \rho_{\mathrm{yz}} \\ \rho_{\mathrm{zx}} & \rho_{\mathrm{zy}} & \rho_{\mathrm{zz}} \end{pmatrix} = \begin{pmatrix} \sigma_{\mathrm{xx}} & \sigma_{\mathrm{xy}} & \sigma_{\mathrm{xz}} \\ \sigma_{\mathrm{yx}} & \sigma_{\mathrm{yy}} & \sigma_{\mathrm{yz}} \\ \sigma_{\mathrm{zx}} & \sigma_{\mathrm{zy}} & \sigma_{\mathrm{zz}} \end{pmatrix}^{-1}$$

$$\rho = \sigma^{-1} = \frac{1}{\det|\sigma|} \begin{pmatrix} \sigma_{\mathrm{yy}}\sigma_{\mathrm{zz}} - \sigma_{\mathrm{yz}}\sigma_{\mathrm{zy}} & \sigma_{\mathrm{xz}}\sigma_{\mathrm{zy}} - \sigma_{\mathrm{xy}}\sigma_{\mathrm{zz}} & \sigma_{\mathrm{xy}}\sigma_{\mathrm{yz}} - \sigma_{\mathrm{xz}}\sigma_{\mathrm{yy}} \\ \sigma_{\mathrm{yz}}\sigma_{\mathrm{zx}} - \sigma_{\mathrm{yx}}\sigma_{\mathrm{zz}} & \sigma_{\mathrm{xx}}\sigma_{\mathrm{zz}} - \sigma_{\mathrm{xz}}\sigma_{\mathrm{zx}} & \sigma_{\mathrm{xz}}\sigma_{\mathrm{yx}} - \sigma_{\mathrm{xx}}\sigma_{\mathrm{yz}} \\ \sigma_{\mathrm{yx}}\sigma_{\mathrm{zy}} - \sigma_{\mathrm{yy}}\sigma_{\mathrm{zx}} & \sigma_{\mathrm{xy}}\sigma_{\mathrm{zx}} - \sigma_{\mathrm{xx}}\sigma_{\mathrm{zy}} & \sigma_{\mathrm{xx}}\sigma_{\mathrm{yy}} - \sigma_{\mathrm{xy}}\sigma_{\mathrm{yx}} \end{pmatrix}$$

It is in general complex to go between conductivity and resistivity!