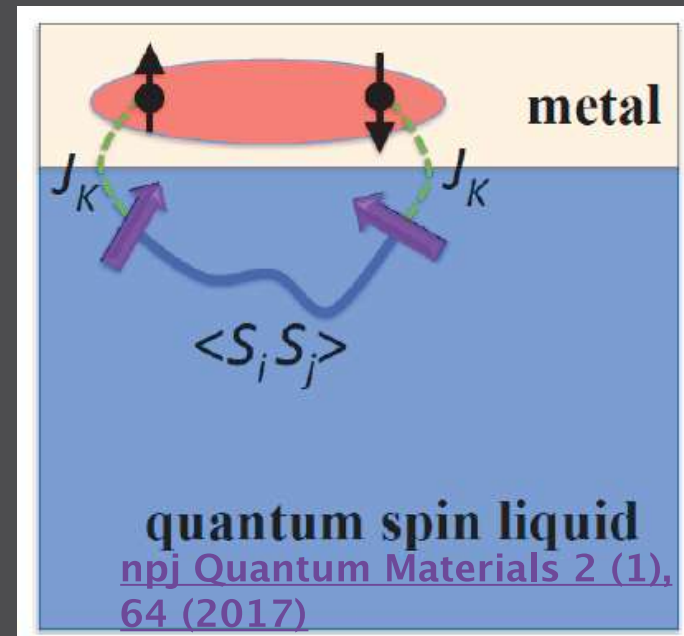
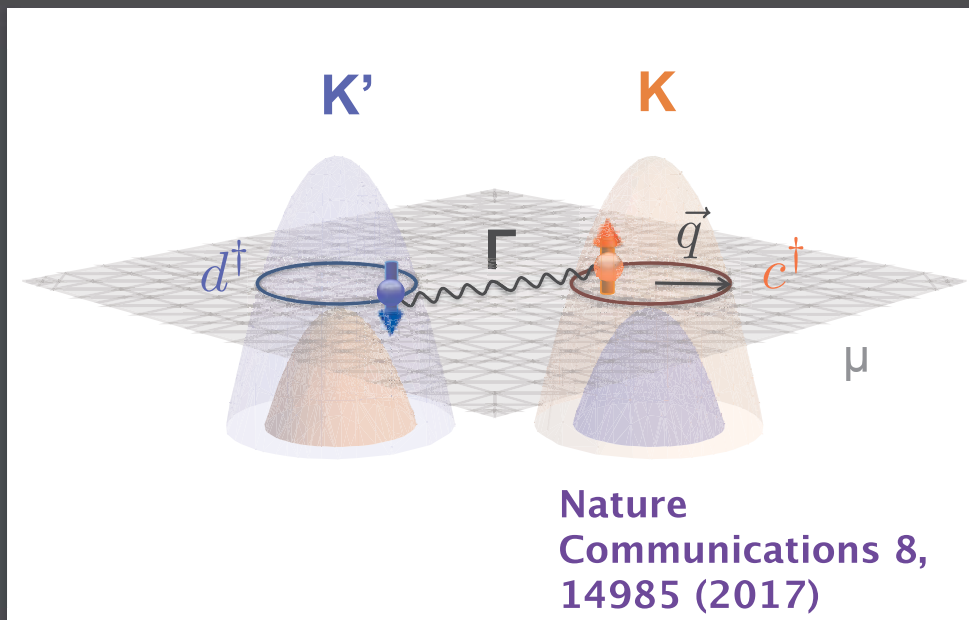


Let There Be Topological Superconductors



Eun-Ah Kim (Cornell)

9.12.2019

Cabrera Summerschool

!! Wanted: Postdocs !!

Bethe/KIC Postdoctoral Fellowship in Theoretical Physics at Cornell's Laboratory of Atomic and Solid State Physics

Cornell University's Laboratory of Atomic and Solid State Physics is soliciting applications for the Bethe/KIC Postdoctoral Fellowship.

This prize fellowship will provide an outstanding theoretical physicist the opportunity to work with theorists and experimentalists in Cornell's physics department. Our group (www.lassp.cornell.edu/people/faculty) has broad interests in hard and soft condensed matter physics, including: cold atom physics, biophysics, statistical physics, hydrodynamics, electronic structure theory, materials science, strongly correlated electrons, nanoscience, computational physics and superconductivity. We also have growing efforts incorporating machine learning into studies of condensed matter physics.

We actively encourage applications from diverse and historically underrepresented candidates.

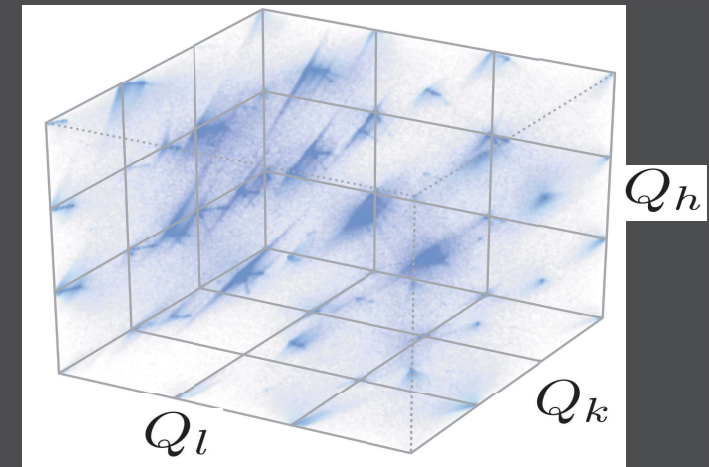
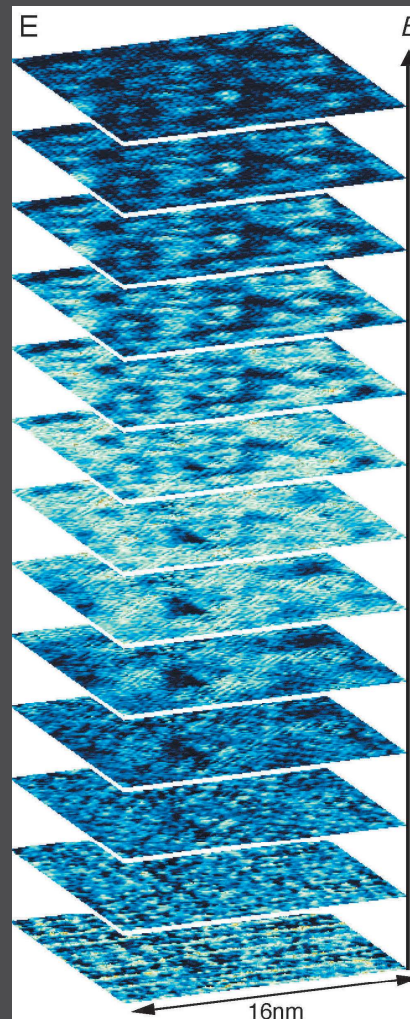
Please submit a CV, publication list, and a 1-3 page research statement to this website at <https://academicjobsonline.org/ajo/jobs/13989>.

<https://academicjobsonline.org/ajo/jobs/13989>

Navigating Data Driven Challenges using Machine Learning

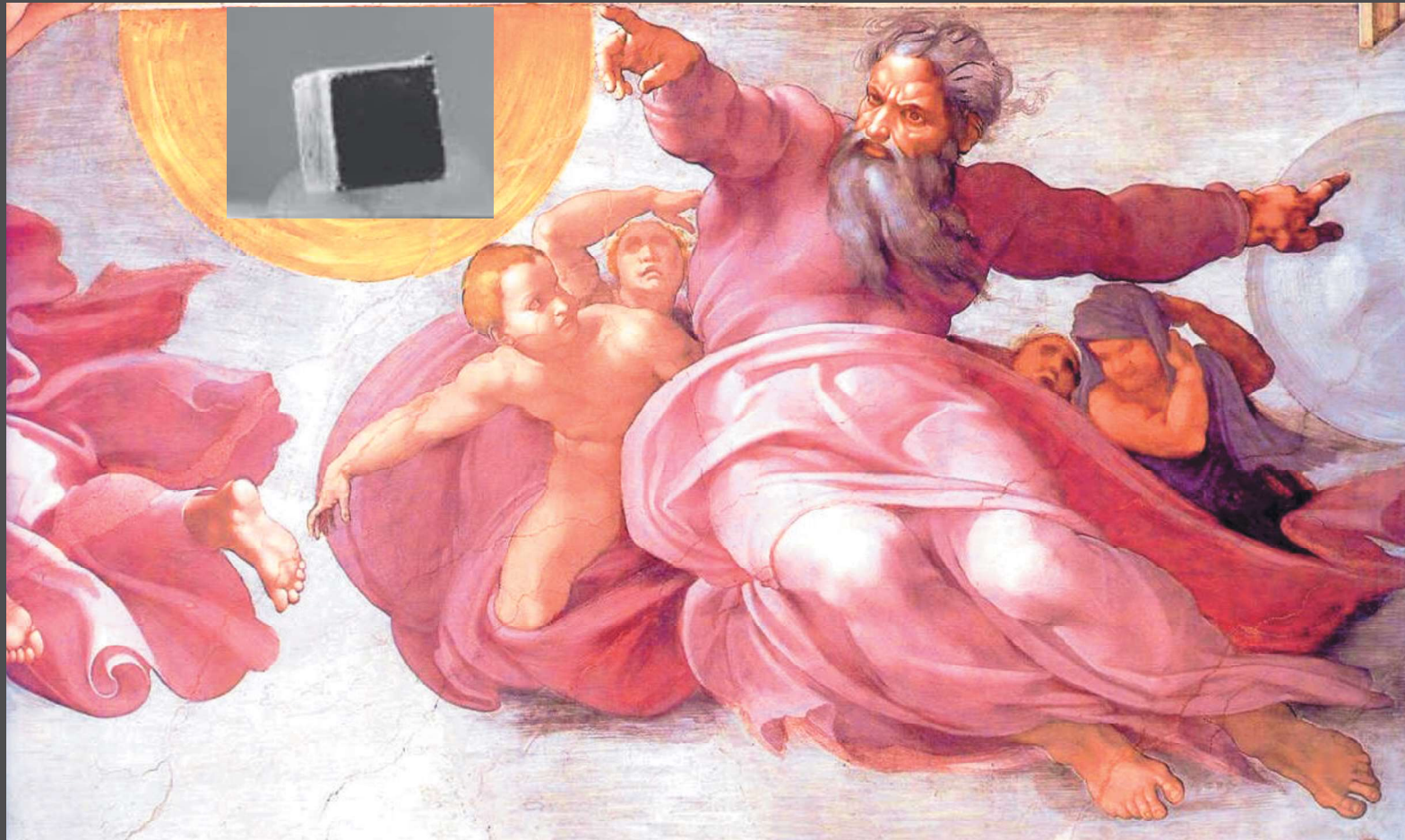


Many-body wave function



Position and
Reciprocal Space Data

“Let there be a topological
superconductor” ...



Odd-parity Superconductors

are Topological
(host Majorana Zero Mode)

Review:

Kallin & Berlinsky, Rep. Prog. Phys. (2016),
Alicea, Rep. Prog. Phys (2012)

Majorana bound state in "spinless" SC

👁 Vortices of p+ip SF → zero modes at the core

Kopnin and Salomaa PRB (1991)

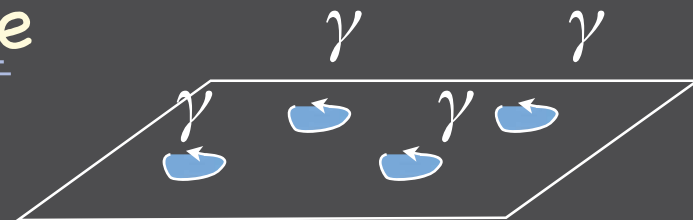
👁 Zero modes are Majorana

▶ BdG qp's $\gamma_i^\dagger = u\psi_i^\dagger + v\psi_i$ $\gamma_i^\dagger(E_n) = \gamma_i(-E_n)$

▶ zero mode: $\gamma_i^\dagger(0) = \gamma_i(0)$

👁 Majorana + vortex composite

➔ non-Abelian Q-bits



Das Sarma, Tewari, Nayak (06)

Stone & Chung(06)

Ivanov(01)

Chung, Bluhm, EAK (07)

3D v.s. 2D

- Statistical angle (topological spin)

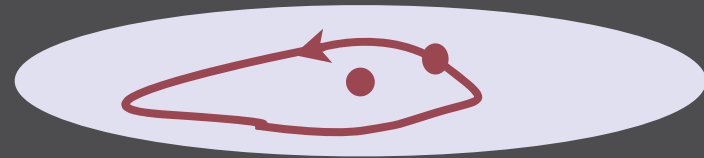
$$\psi(r_1, r_2) = e^{i\theta} \psi(r_2, r_1)$$

- 3D



- ▶ double exchange = I
- ▶ $\theta = 0$ (boson), π (fermion)

- 2D



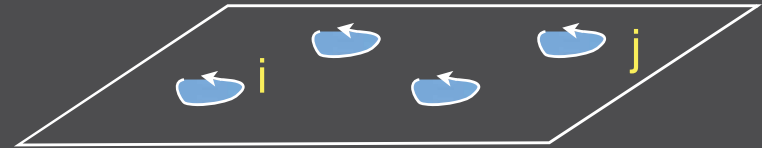
- ▶ θ can be arbitrary
- ▶ (abelian) Anyon

A pair of MBS = Q-bit

👁 The “fusion” rule and Q-bits

$$c = \gamma_i + i\gamma_j$$

$$c^\dagger = \gamma_i - i\gamma_j$$



: each pair of MBS host a two-state system

👁 $2n$ MBS's

$N_{2n} = 2^{n-1}$ dimensional Hilbert space

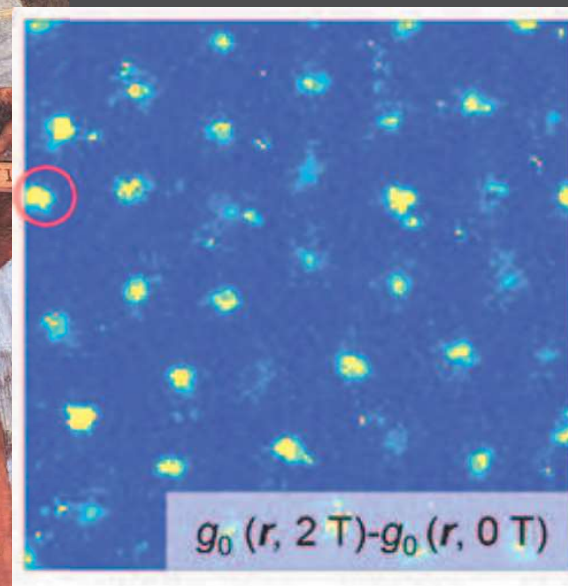
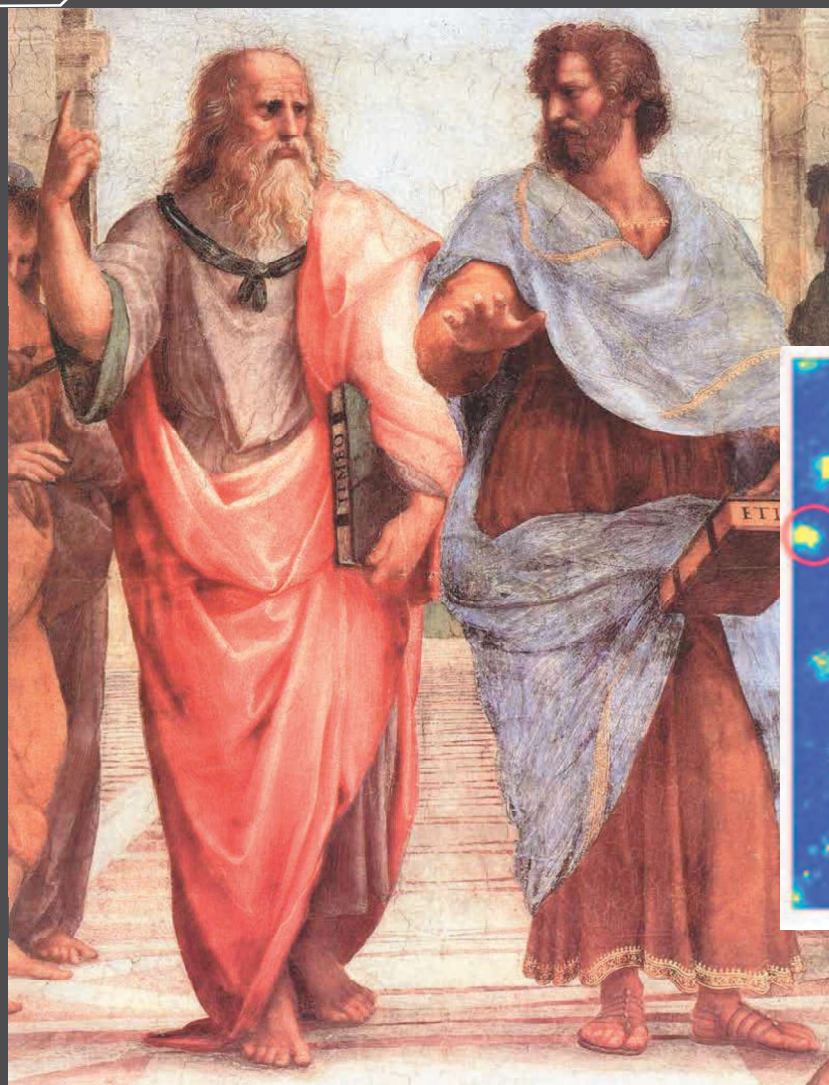
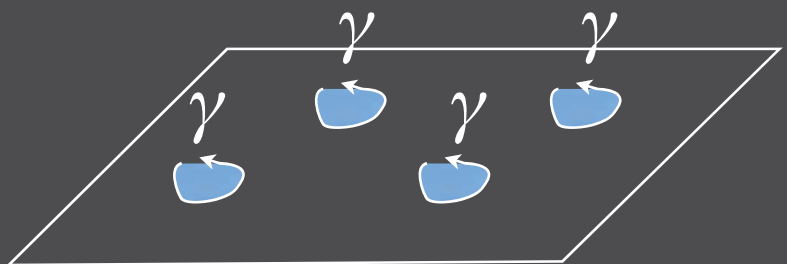
Non-Abelian Statistics: Gates

- n- nonabelian qp state \mapsto set of Q-bits

$$\Psi(x_1, \dots, x_n) = \begin{pmatrix} \psi_1 \\ \vdots \\ \psi_{\underline{\underline{d(n)}}} \end{pmatrix} \rightarrow \begin{array}{l} \text{exchange of qp's: rotation} \\ \text{in } d(n) \text{ dim} \\ \text{Hilbert space} \end{array}$$

$$\Psi(x_1 \leftrightarrow x_3) = \underline{\underline{M}} \Psi(x_1, \dots, x_n)$$

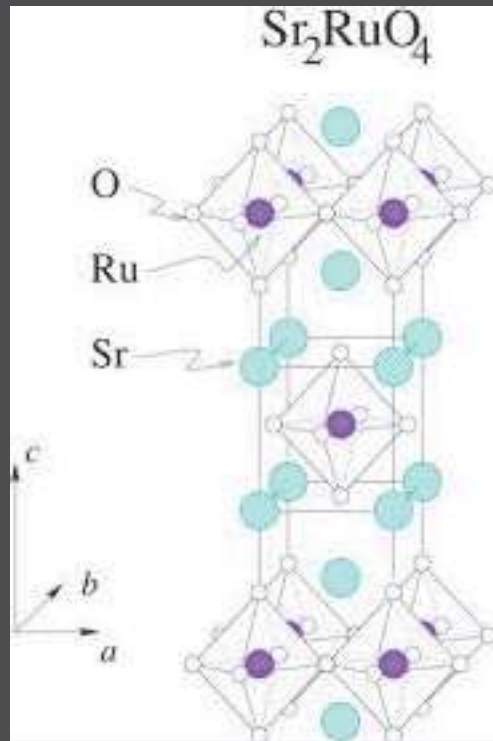
$$\Psi(x_1 \leftrightarrow x_2) = \underline{\underline{N}} \Psi(x_1, \dots, x_n)$$



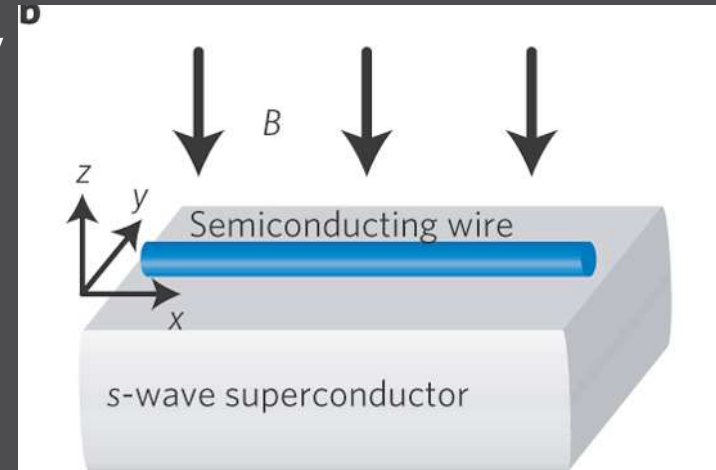
Hai-Hu Wen et al
(1909.01686)

Q. Topological Superconductor material?

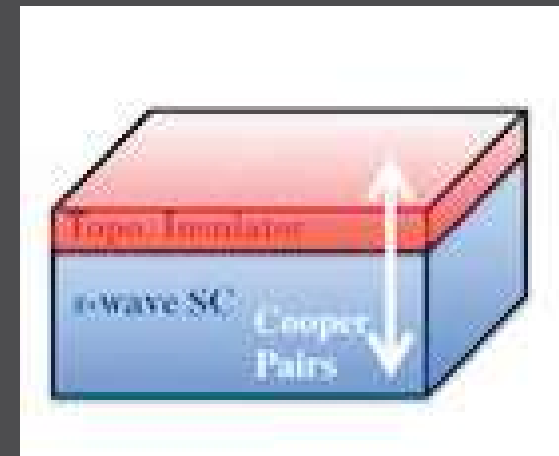
Bulk



1D proximity



2D proximity?



Review:

Kallin & Berlinsky, Rep. Prog. Phys. (2016),
Alicea, Rep. Prog. Phys (2012)

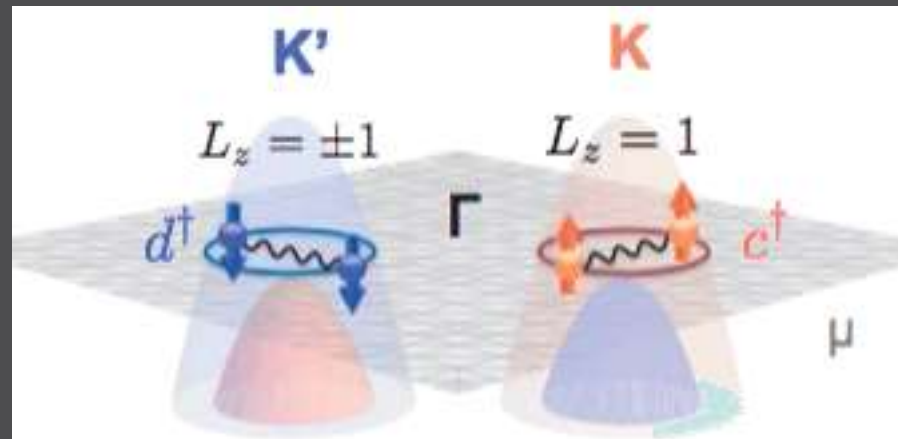
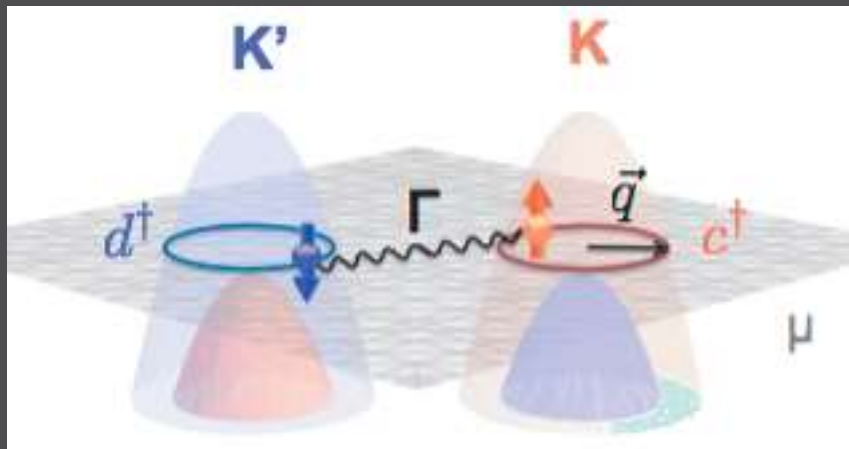
Designing 2D topological SC's

- 2D topological SC
 - odd-parity SC of spinless fermions
 - Majorana bound state
- Strategies:
 - 1) spinlessness
 - 2) interaction

Strategy I

Manipulate the band
structure

Topological superconductivity in group-VI TMDs



Yi-Ting Hsu, Abolhassan Vaezi, Mark Fischer, E-AK (Nature Communications 8, 14985 (2017))



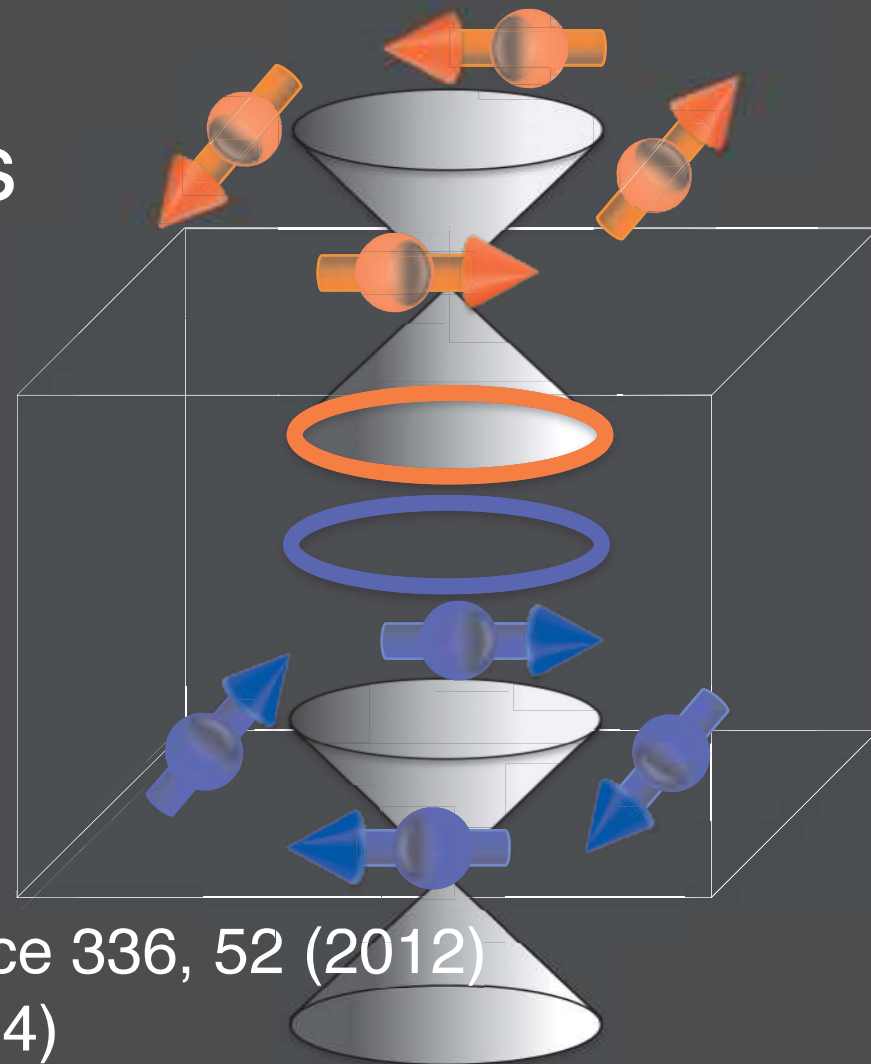
Yi-Ting Hsu



Abolhassan Vaezi

Spinless fermion via **real space** splitting

- TI surface states
- Proximity induce topo SC

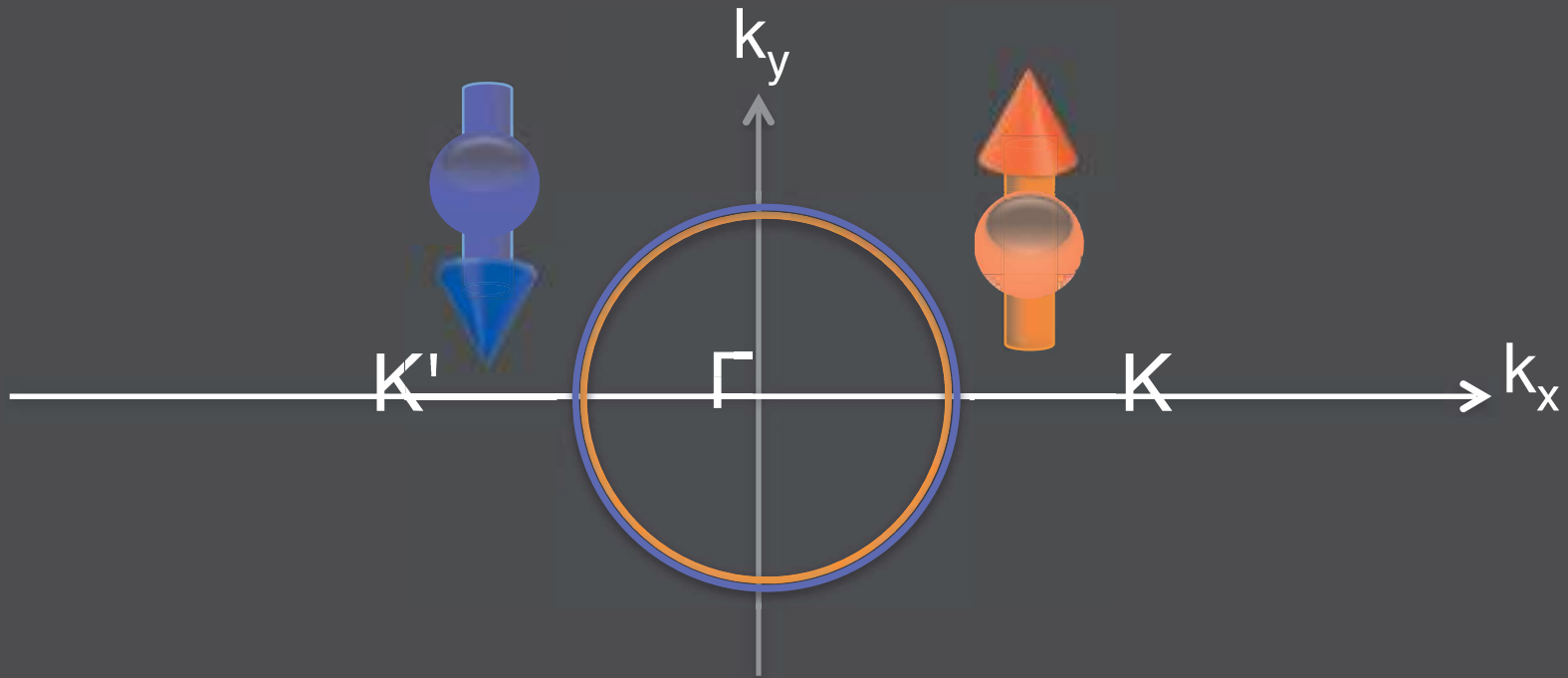


Fu & Kane, PRL (2008)

Experiments: Wang et al Science 336, 52 (2012)

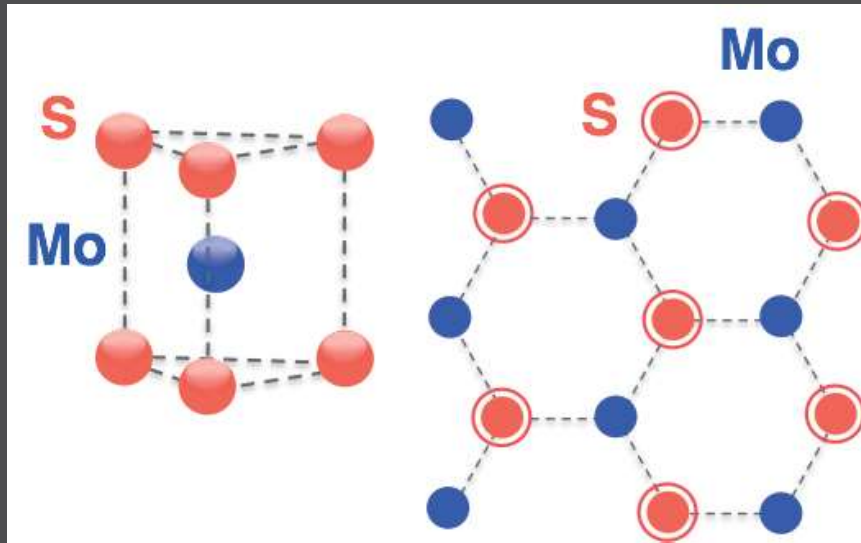
Xu et al, Nat.Phys 10, 943 (2014)

Spinless fermion via **k-space** splitting?



Monolayer group VI TMD's

MoS_2 , WS_2 , MoSe_2 , WSe_2



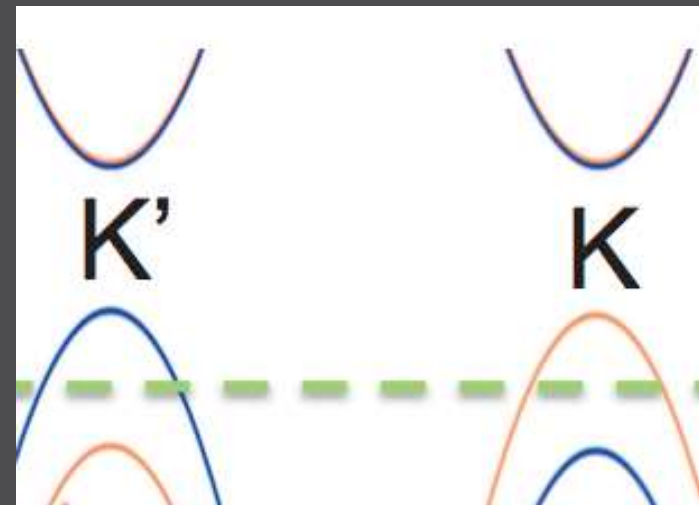
- Non-centro symmetric
 - ➔ Direct Gap $\sim 2\text{eV}$
 - ➔ Dresselhaus spin-orbit

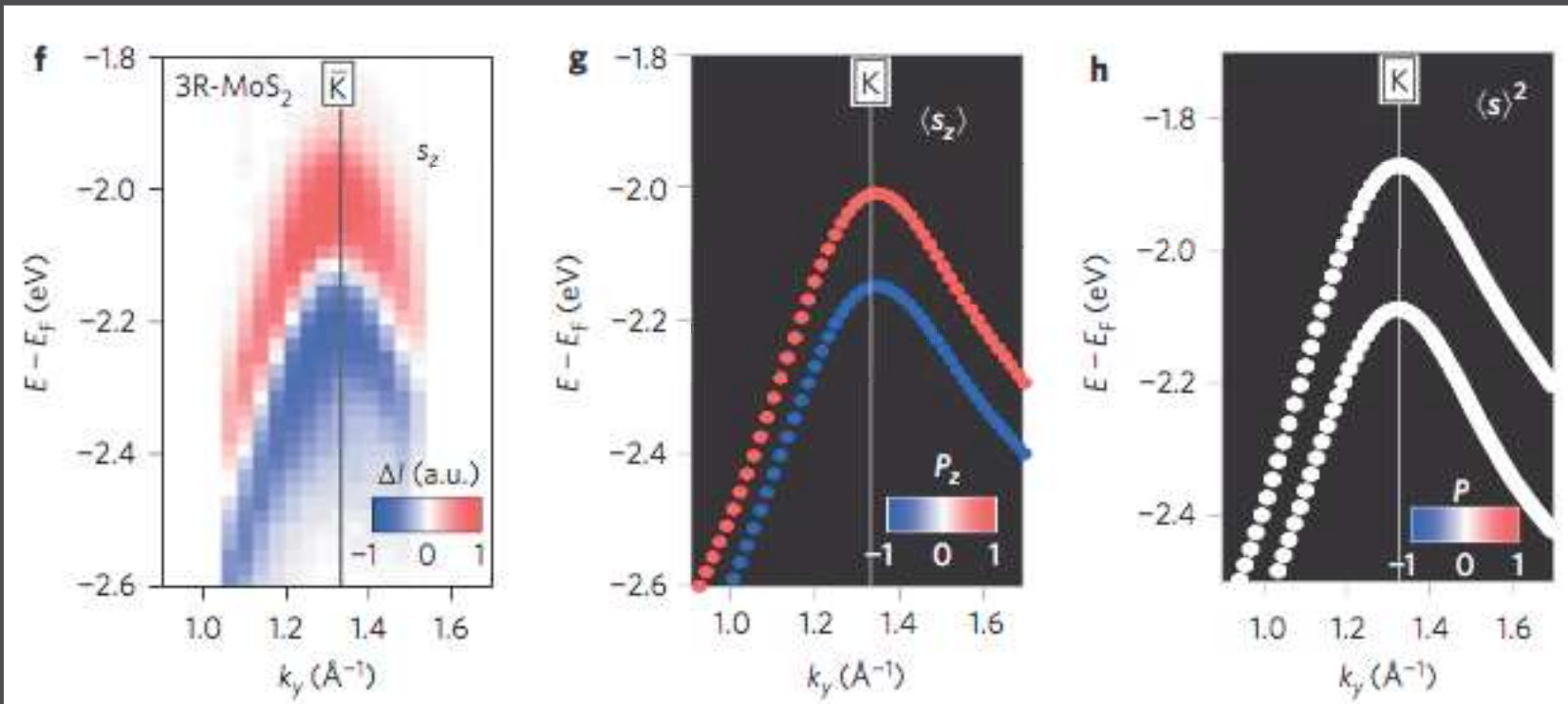
Band-selective spin-splitting

- Partially filled crystal-field-split d-bands
 - Conduction band $|d_{z^2}\rangle: l_z=0$
 - Valence band $\frac{1}{\sqrt{2}}(|d_{x^2-y^2}\rangle \mp i|d_{xy}\rangle) : l_z = \mp 1$

- Spin-orbit coupling $\vec{L} \cdot \vec{S}$

150~460meV

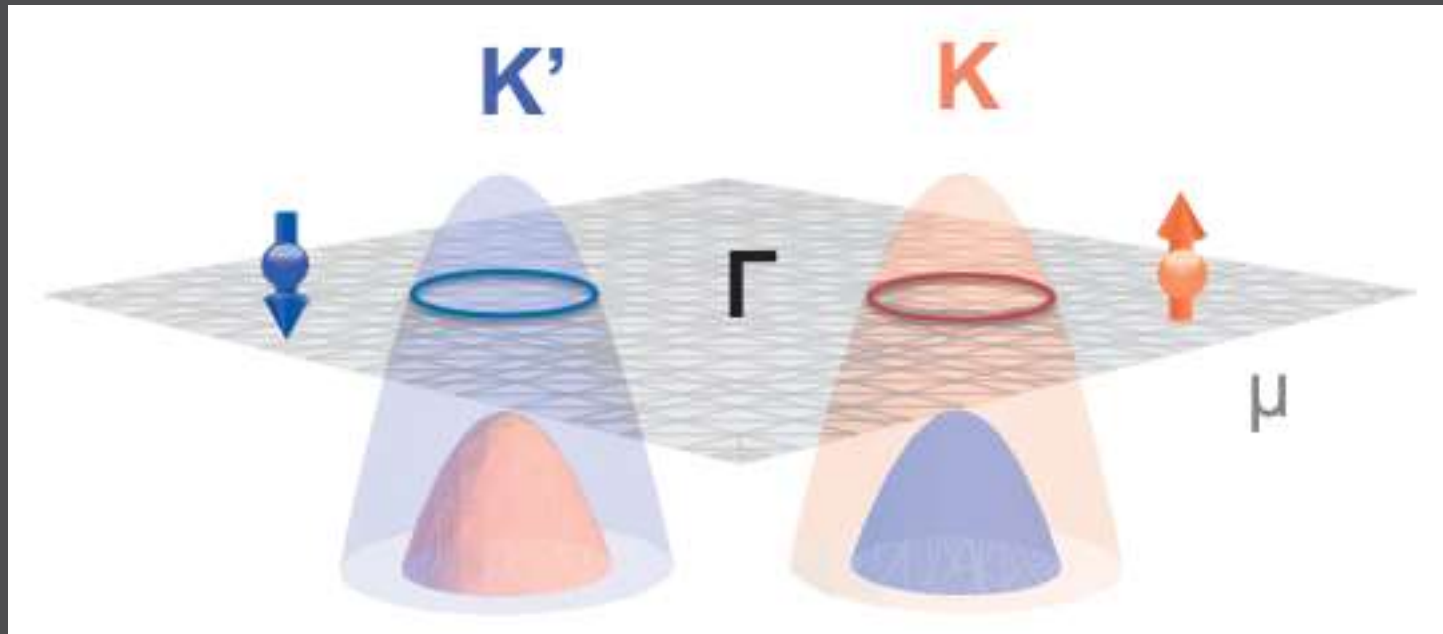




Iwasa group N. Nano (2014)

k-space spin-split FS?

p-doped group VI- TMD!

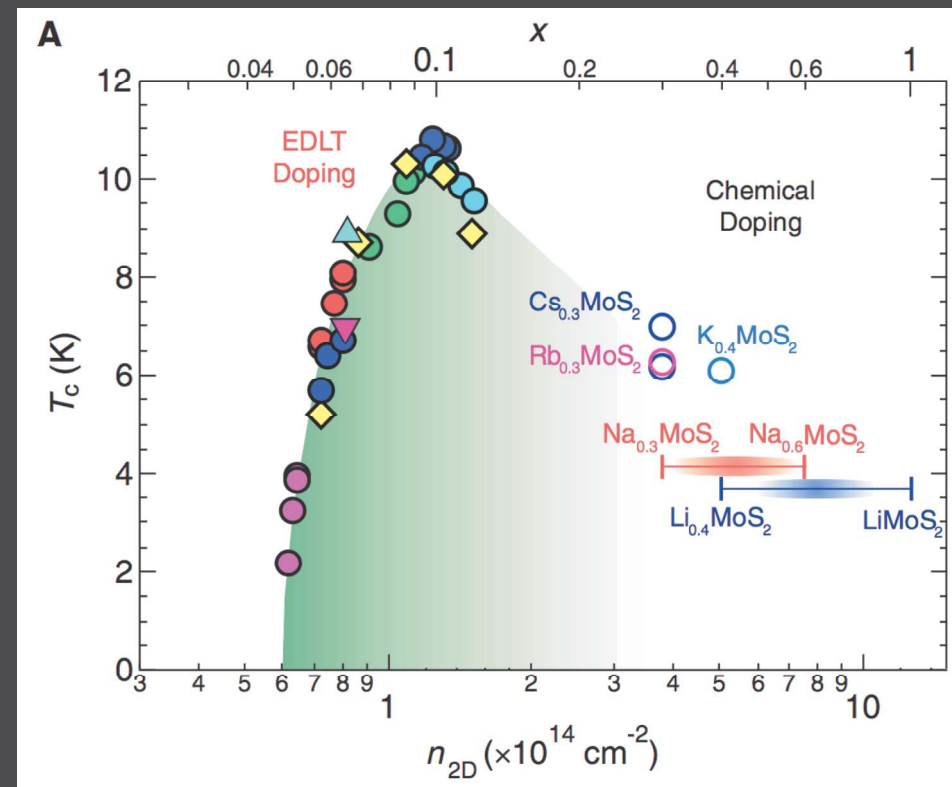


Juice for superconductivity?

- d electrons => expect correlation effects

- n-doped

J.T.Ye et al. (Science 2012)



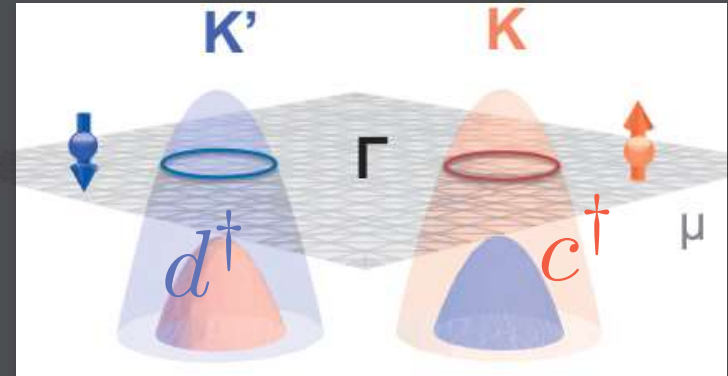
p-doped TMD

k-space spin-split Fermi surfaces
+
Moderate correlation (d-electron)



Topological SC?

Model



- Kinetic term

$$H_0(\vec{q}) = \sum_{\vec{q}} \epsilon(\vec{q}) (c_{\vec{q}}^\dagger c_{\vec{q}} + d_{\vec{q}}^\dagger d_{\vec{q}})$$

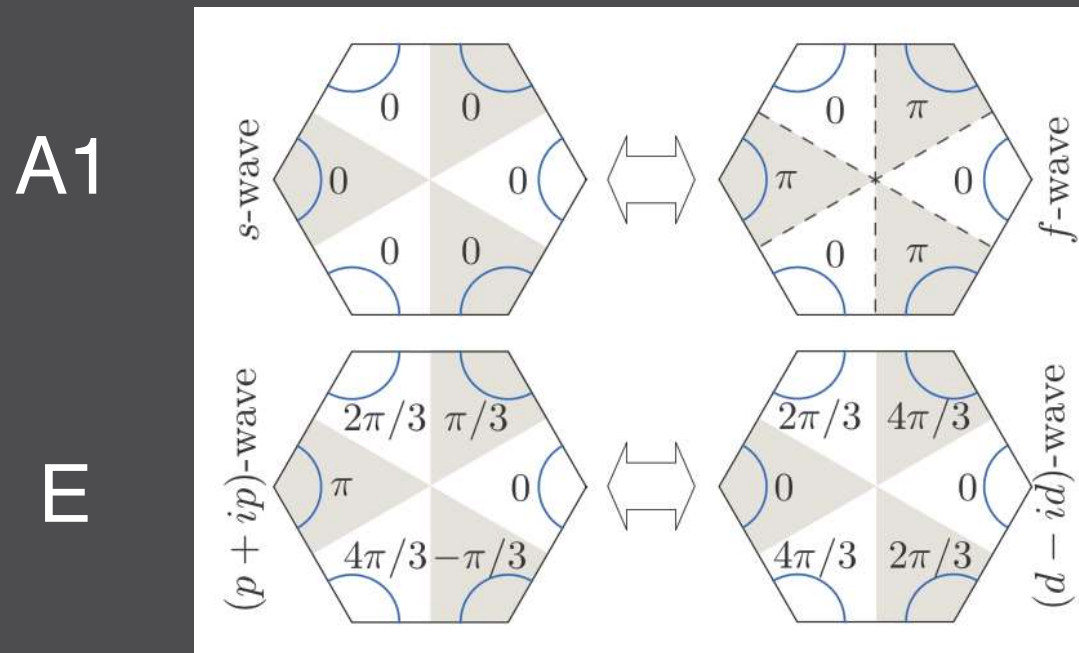
- Repulsive interaction term

$$H'(W) = \sum_i U n_{i,\uparrow} n_{i,\downarrow}$$

- Point group C3v

Implication of spin-valley locking

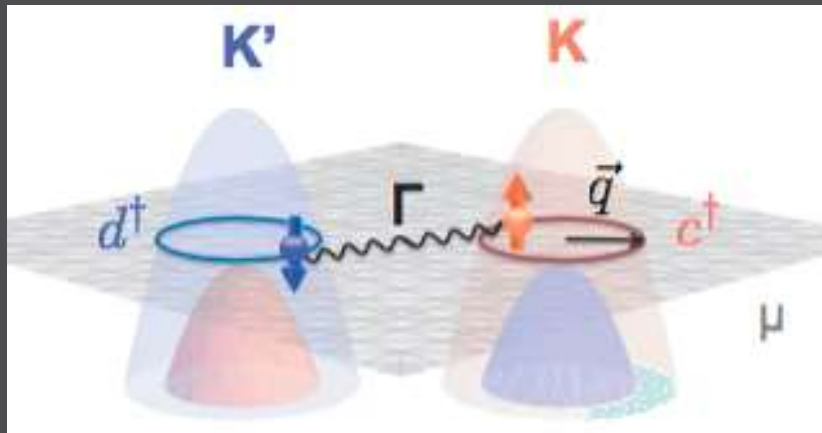
- Irrep's of C_{3v} (with full gap)



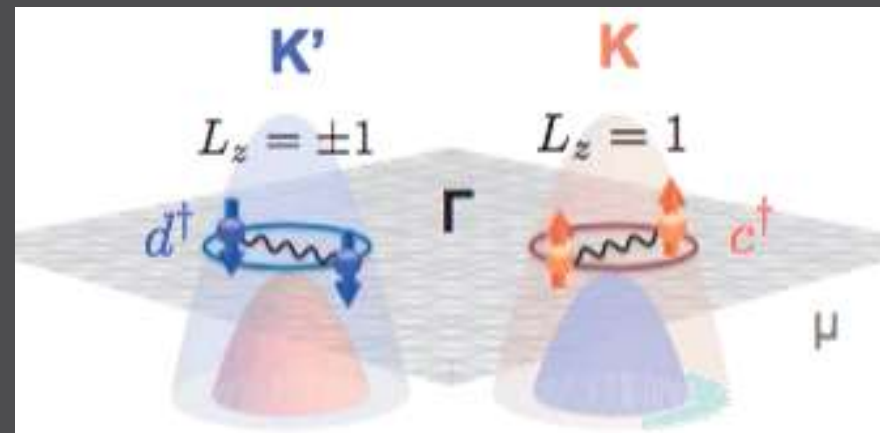
- If s-wave is blocked, f-wave is blocked!!

Two possibilities

- Intra-pocket p+ip
- Inter-pocket p'wave



- T-breaking
- C=2

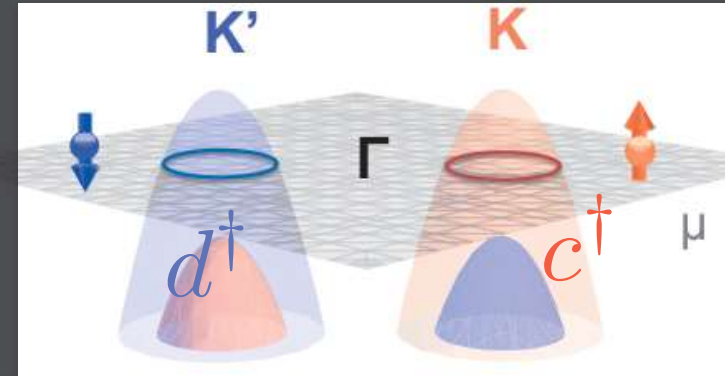


- Modulated
- C= ± 1 per pocket

Two-step RG on p-doped TMD

Following Raghu, Kivelson, Scalapino (PRB 2010)

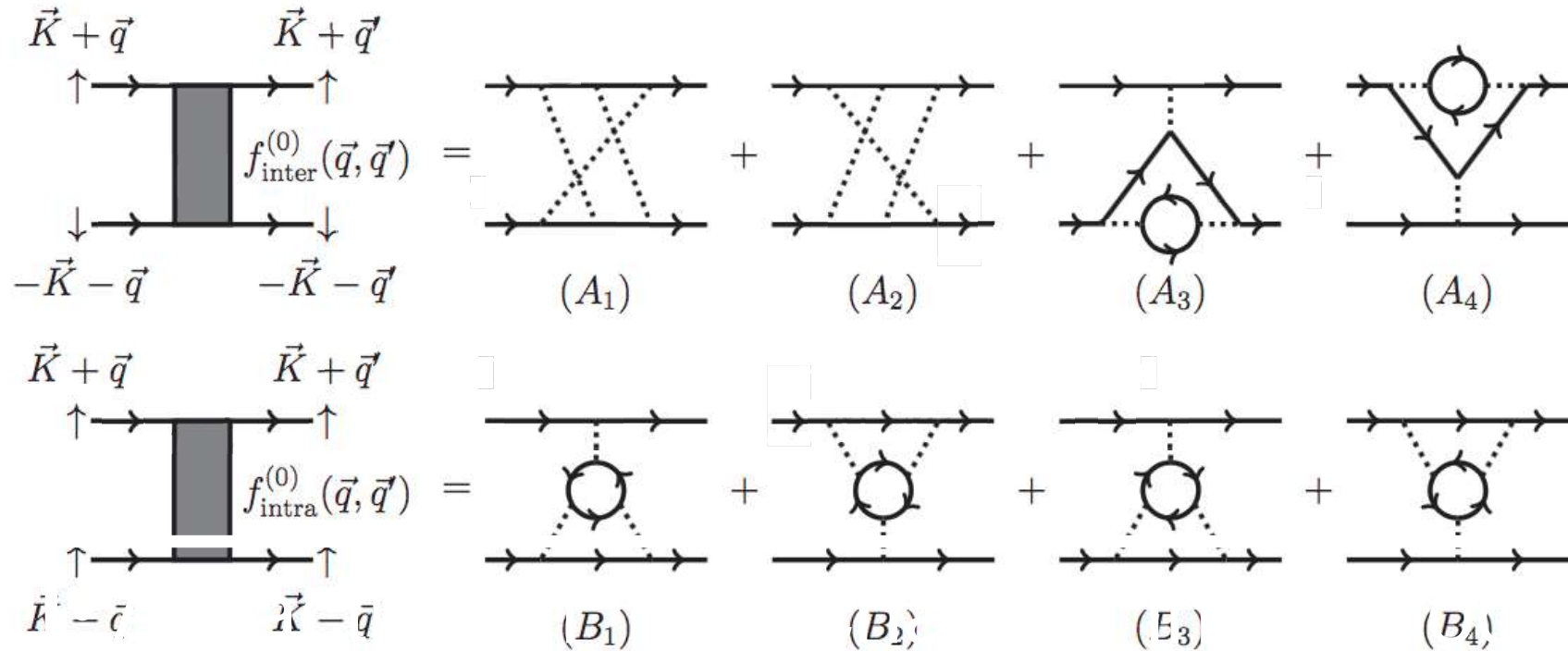
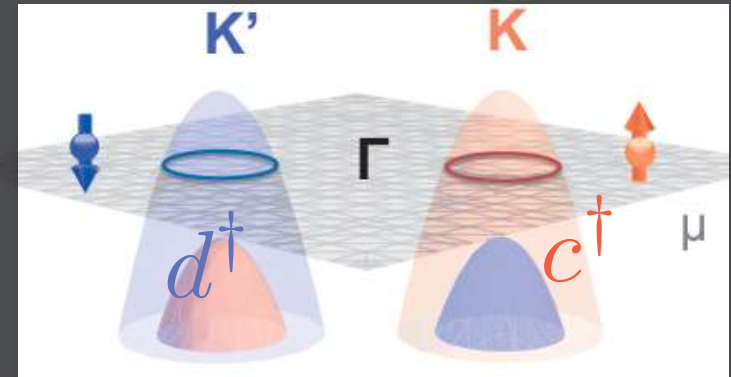
Step I: $W \rightarrow \Lambda_0$



- At scale W : Microscopic model
- At scale Λ_0 : Effective model

$$H'_{eff}(\Lambda_0) = \sum_{\vec{q}, \vec{q}'} g_{inter}^{(0)}(\vec{q}, \vec{q}') c_{\vec{q}'}^\dagger d_{-\vec{q}'}^\dagger d_{-\vec{q}} c_{\vec{q}} + g_{intra}^{(0)}(\vec{q}, \vec{q}') d_{\vec{q}'}^\dagger d_{-\vec{q}'}^\dagger d_{-\vec{q}} d_{\vec{q}} + (c \leftrightarrow d)$$

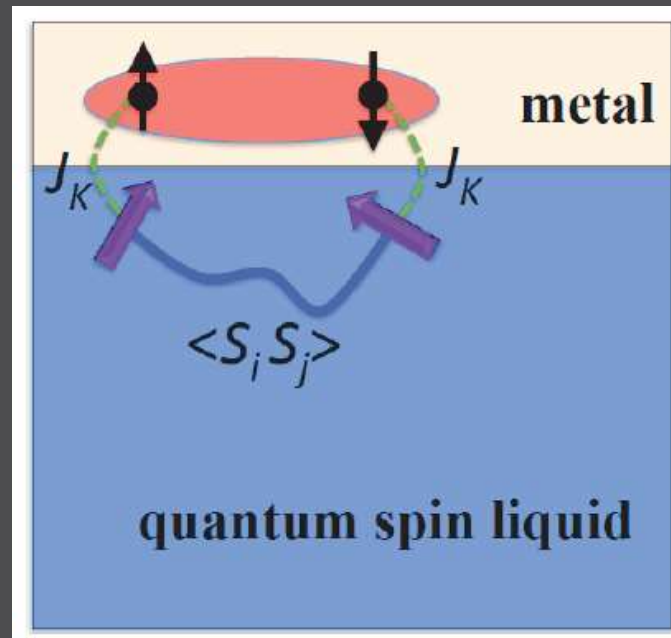
Step I: $W \rightarrow \Lambda_0$



Strategy II

- Manipulate **the pairing interaction**:
target non-phononic mechanism

Topological Superconductivity in Metal/ Quantum-Spin-Ice Heterostructures



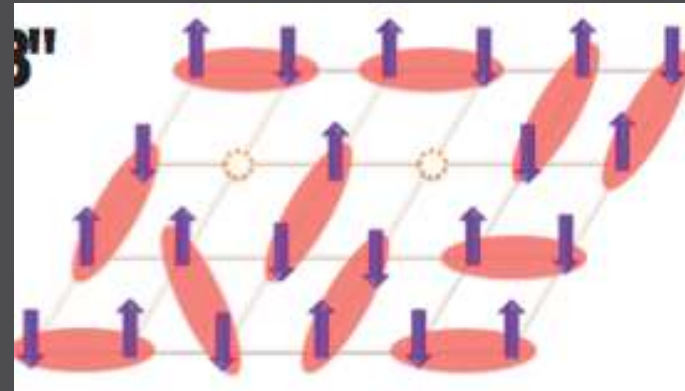
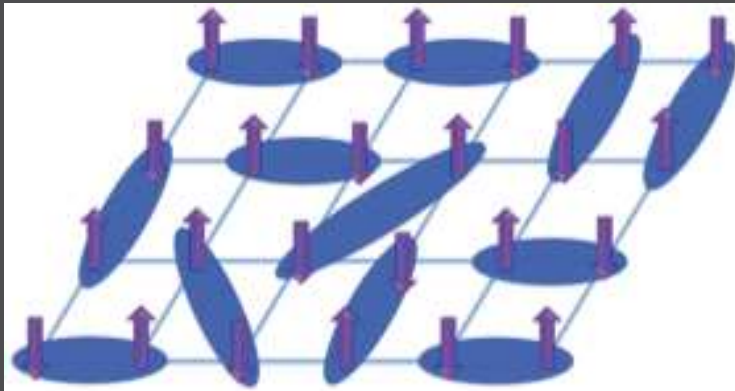
Jian-Huang She, Choonghyun Kim, Craig Fennie, Michael Lawler, E-AK (npj Quantum Materials 2 (1), 64 (2017))

Wanted: non-phononic mechanism

Dope a Quantum spin liquid



P.W. Anderson



RVB singlet

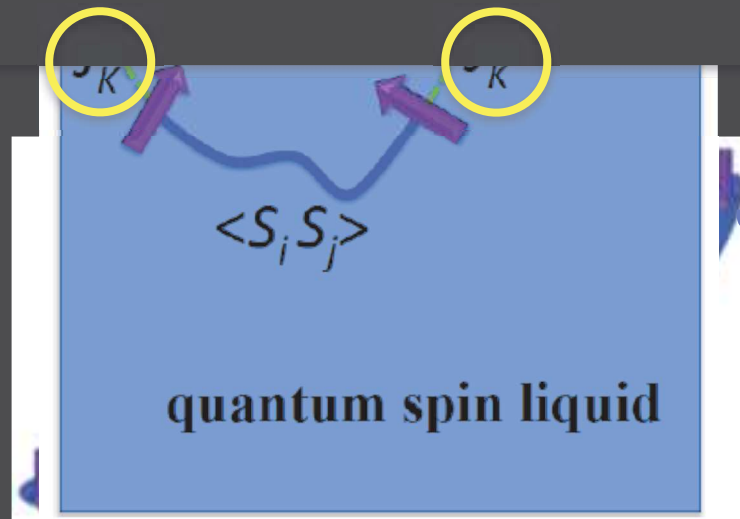


Cooper pair singlet

Wanted: non-phononic mechanism



Use Quantum paramagnet



E_F

E_F, J_{ex}, J_K

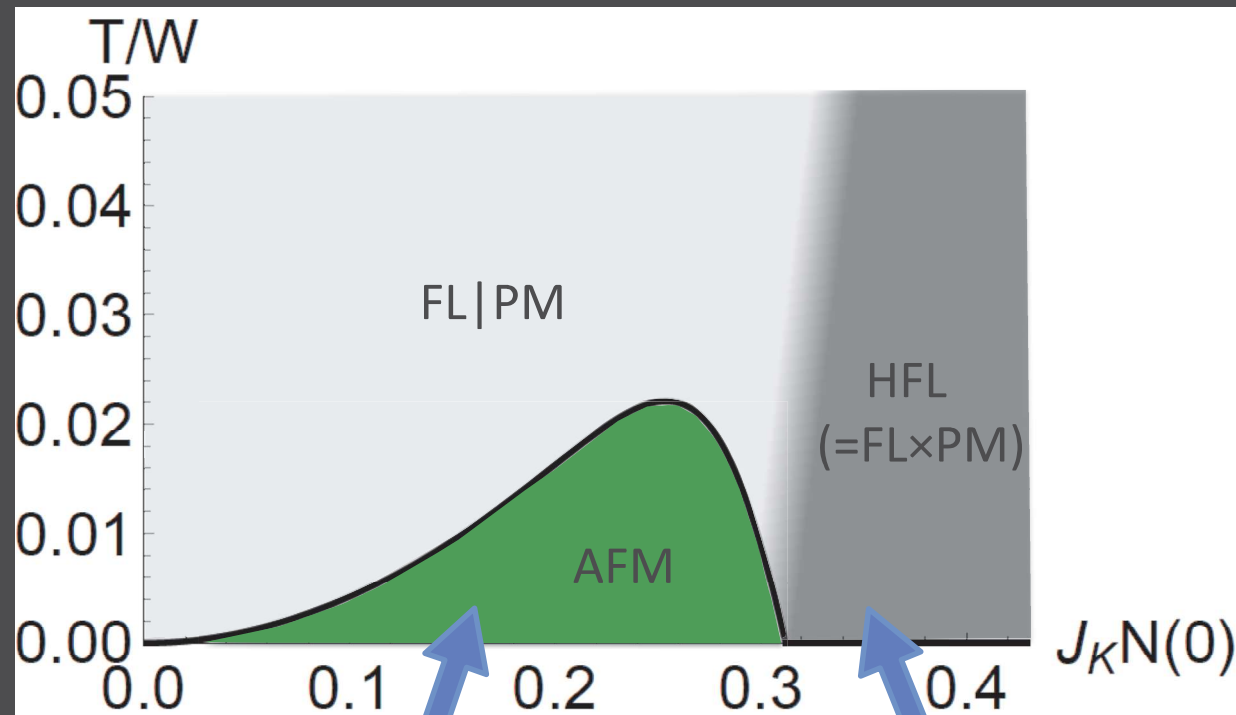
- Characteristic energy scales:

- Perturbative limit:

$$J_K / E_F \ll 1$$

- Spin-fermion model

Spin-fermion model for $J_{\text{ex}}=0$



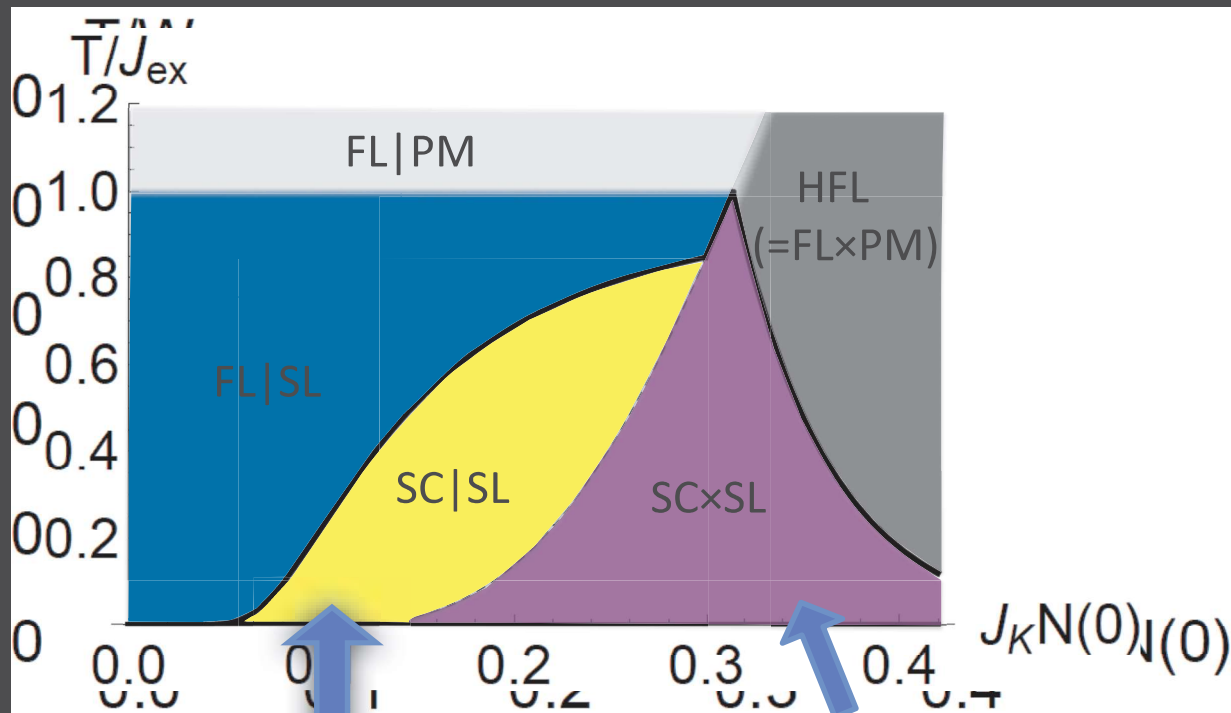
RKKY interaction

Kondo-Singlet

Doniach (1977)

Spin-fermion model for $J_{ex} + \text{Frustration}$

For $J_{RKKY} \sim J_K^2 N(0) < J_{ex}$ no AFM



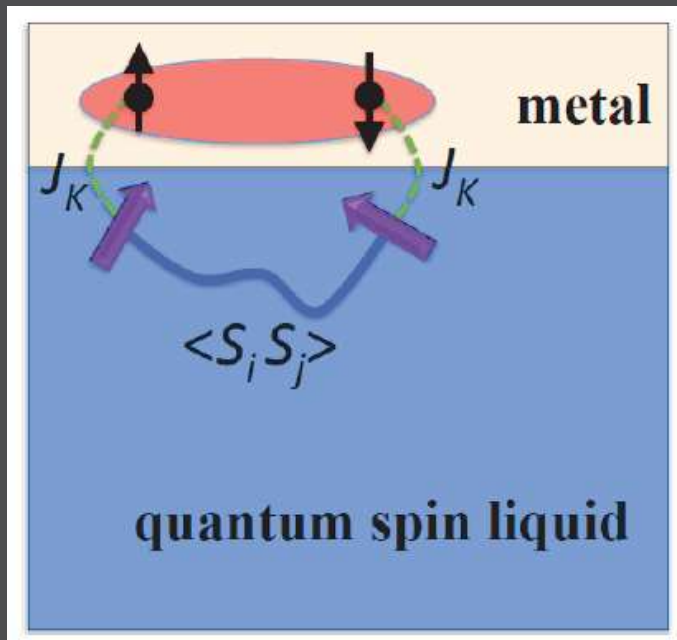
SC “riding” on
QSL

Kondo-Singlet + RVB
singlet+Cooper pair

Coleman & Andrei (1989)

Senthil, Vojta, Sachdev (2003)

How to predictively materialize SCIQPM ?

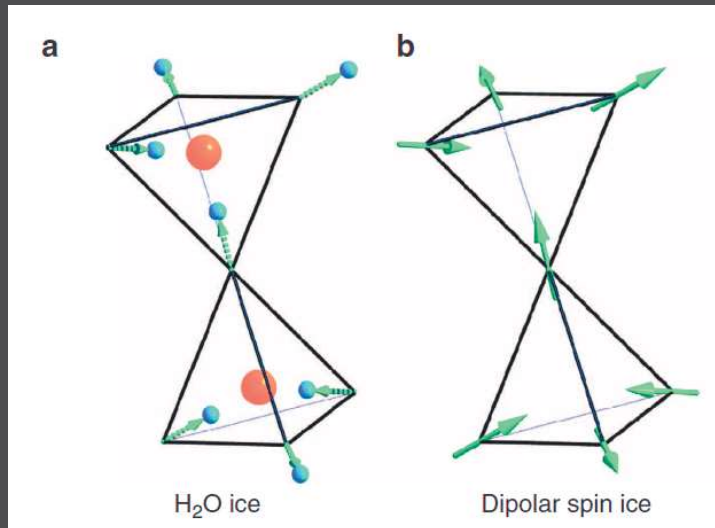


Simple isotropic metal

1. $\langle S \rangle = 0$
2. Dynamic spin fluctuation
 $\langle S_i S_j \rangle$
3. Well understood

➔ Quantum Spin Ice

Emergent Vector Field in Spin Ice



Kimura et al (2013)

- Vector Field Propagator
- Spin-spin correlation

- 2-in 2-out ice rule

$$\nabla \cdot \vec{S}(\mathbf{r}) = 0$$

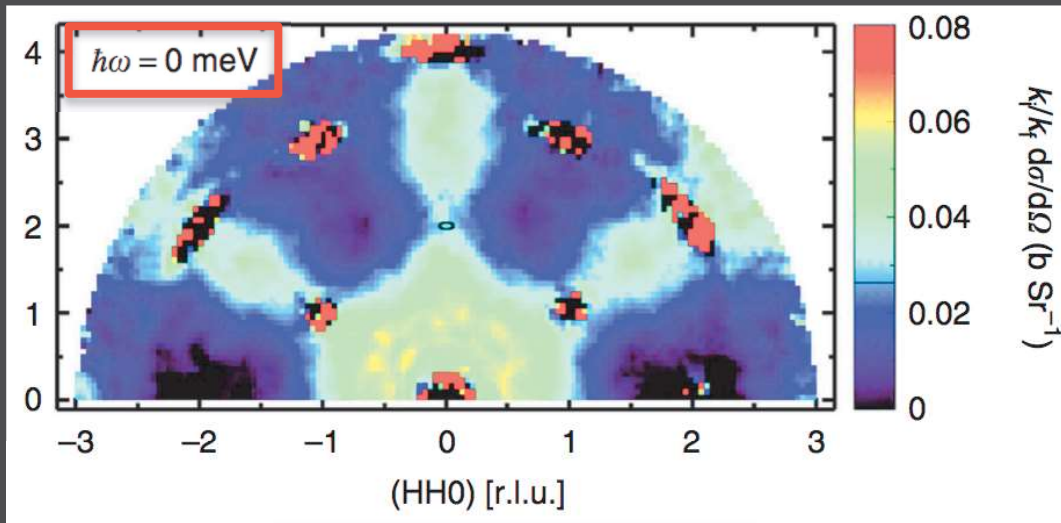
$$\vec{S}(\mathbf{r}) = \nabla \times \vec{A}(\mathbf{r})$$

$$\langle A_a(\mathbf{q}) A_b(-\mathbf{q}) \rangle \sim \frac{1}{q^2} (\delta_{ab} - 2\hat{q}_a \hat{q}_b)$$

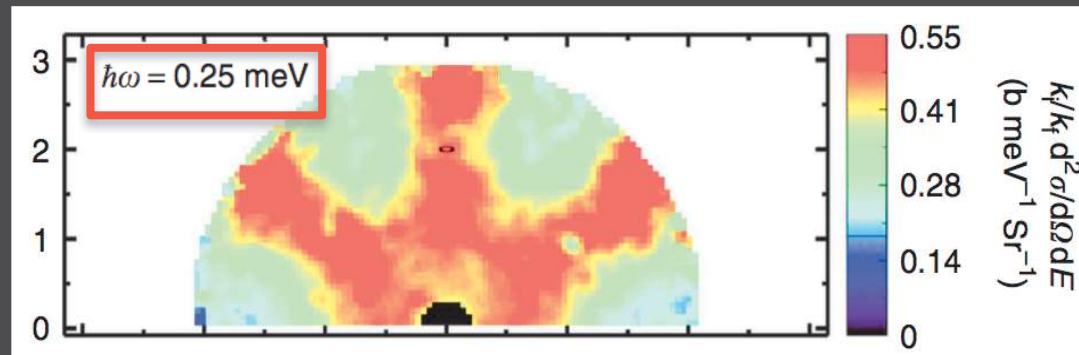
$$\langle S_a(\mathbf{q}) S_b(-\mathbf{q}) \rangle \sim \delta_{ab} - \hat{q}_a \hat{q}_b$$

Quantum fluctuations in spin-ice-like $\text{Pr}_2\text{Zr}_2\text{O}_7$

K. Kimura¹, S. Nakatsuji^{1,2}, J.-J. Wen³, C. Broholm^{3,4,5}, M.B. Stone⁵, E. Nishibori⁶ & H. Sawa⁶



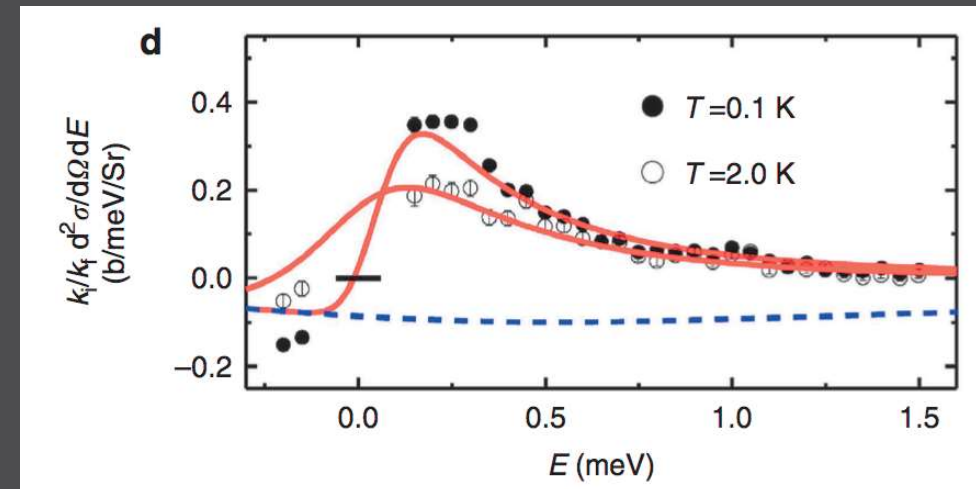
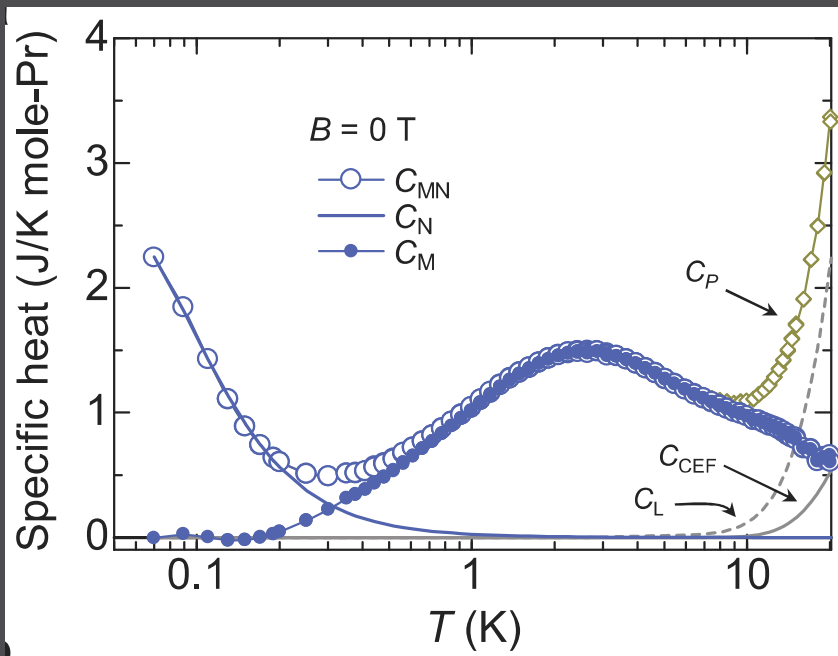
- Elastic neutron: pinch points (spin-ice like)



- Inelastic neutron: over 90% weight

Quantum fluctuations in spin-ice-like $\text{Pr}_2\text{Zr}_2\text{O}_7$

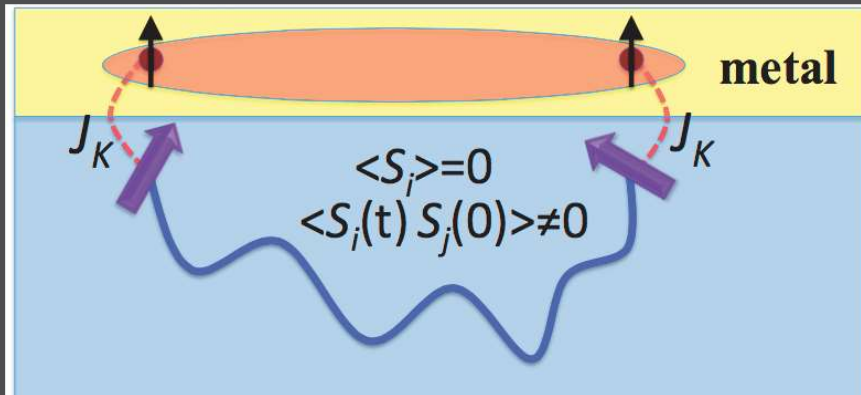
K. Kimura¹, S. Nakatsuji^{1,2}, J.-J. Wen³, C. Broholm^{3,4,5}, M.B. Stone⁵, E. Nishibori⁶ & H. Sawa⁶



- No order down to 20mK

- Spin fluctuation scale $\omega_s = 0.17$ meV

Effective Continuum Theory



$$H_c = \sum_{\mathbf{k}\alpha} \left(\frac{\hbar^2 k^2}{2m} - E_F \right) \psi_\alpha^\dagger(\mathbf{k}) \psi_\alpha(\mathbf{k})$$

$$H_K(t) = J_K v_{\text{cell}} \sum_{\alpha\beta} \int d^2\mathbf{r} \psi_\alpha^\dagger(\mathbf{r}) \sigma_{\alpha\beta}^a \psi_\beta(\mathbf{r}) S_a(\mathbf{r}_\perp = \mathbf{r}, z = 0, t)$$

- Integrate out spins \gg Effective e-e interaction

$$H_{\text{int}}(t) = -(J_K^2 v_{\text{cell}}^2 / 2\hbar) \sum_{ab} \int dt' \int d^2\mathbf{r} d^2\mathbf{r}' s_a(\mathbf{r}, t) \langle S_a(\mathbf{r}, 0, t) S_b(\mathbf{r}', 0, t') \rangle s_b(\mathbf{r}', t')$$

$$s_a(\mathbf{r}, t) = \sum_{\alpha\beta} \psi_\alpha^\dagger(\mathbf{r}, t) \sigma_{\alpha\beta}^a \psi_\beta(\mathbf{r}, t)$$

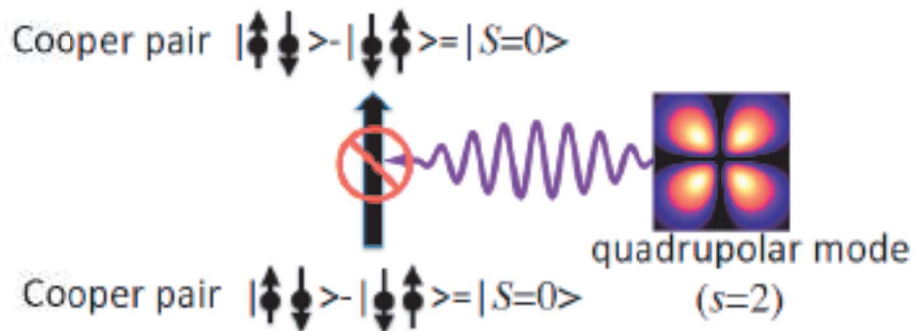
Selection Rule Dictated Odd-Parity

- Pair binding problem with dipole-dipole interaction

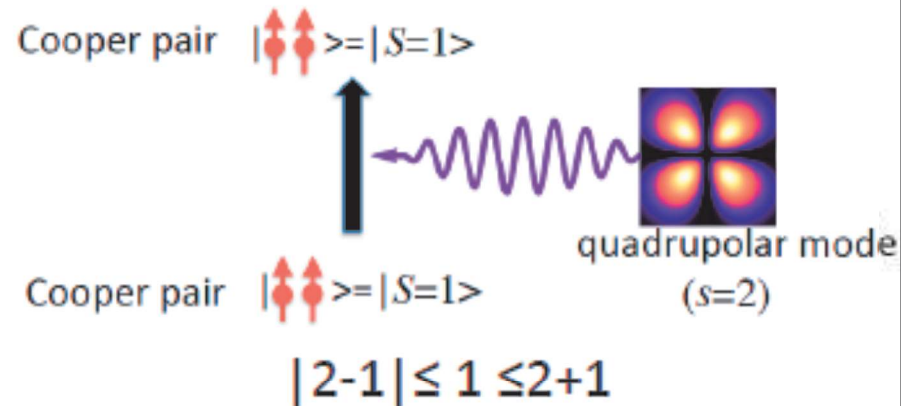
$$V_{\text{dd}} = \frac{1}{r^3} [\vec{S}_1 \cdot \vec{S}_2 - 3(\vec{S}_1 \cdot \hat{r})(\vec{S}_2 \cdot \hat{r})] \propto \mathcal{R}^{(2)}(\mathbf{r}_1, \mathbf{r}_2) \cdot \mathcal{S}^{(2)}(\mathbf{s}_1, \mathbf{s}_2)$$

- Wigner-Eckart thm: $\langle l' | \mathcal{T}^{(r)} | l \rangle = 0$ unless $|r - l| \leq l' \leq (r + l)$

C



D



Dealing with **interacting** electrons?

$$H_{\text{int}}(t) = -(J_K^2 v_{\text{cell}}^2 / 2\hbar) \sum_{ab} \int dt' \int d^2\mathbf{r} d^2\mathbf{r}' s_a(\mathbf{r}, t) \langle S_a(\mathbf{r}, 0, t) S_b(\mathbf{r}', 0, t') \rangle s_b(\mathbf{r}', t')$$

- Separation of scale: $\omega_s / E_F \ll 1$

→ “Migdal theorem”

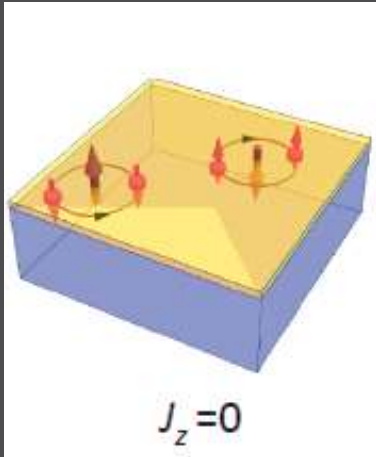
- Dimensionless ratio: $\lambda \sim N(0)V \sim J_K^2 N(0) / J_{\text{ex}} < 1$

- Full problem \approx

solving the **BCS mean-field**

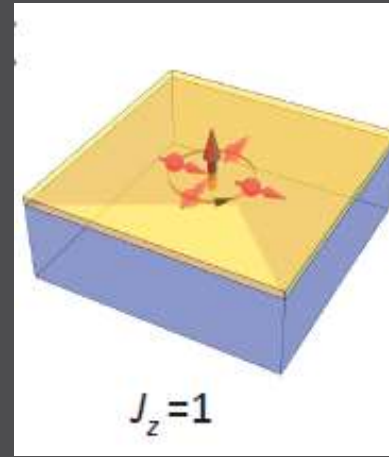
theory $T_c \sim \omega_s e^{-1/\lambda}$

Leading channels



$$(k_x + ik_y) | \downarrow \downarrow \rangle$$

$$+ (k_x - ik_y) | \uparrow \uparrow \rangle$$



$$(k_x \pm ik_y) \frac{ | \uparrow \downarrow \rangle + | \downarrow \uparrow \rangle }{ \sqrt{2} }$$

