



# Broken Paradigms

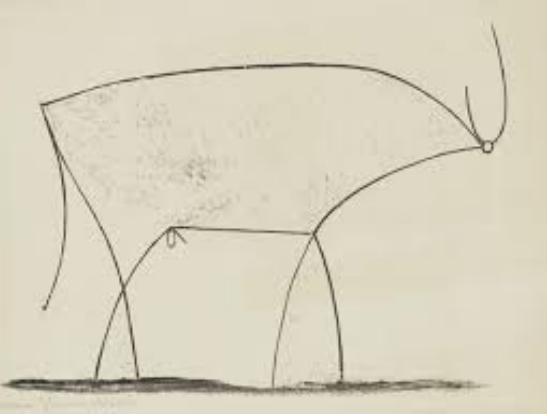
- Band Insulator to superconductor: Intervening bosonic phases
- Disorder-driven Superconductor-Insulator Transition
- Magnetic Field driven SIT

Nandini Trivedi  
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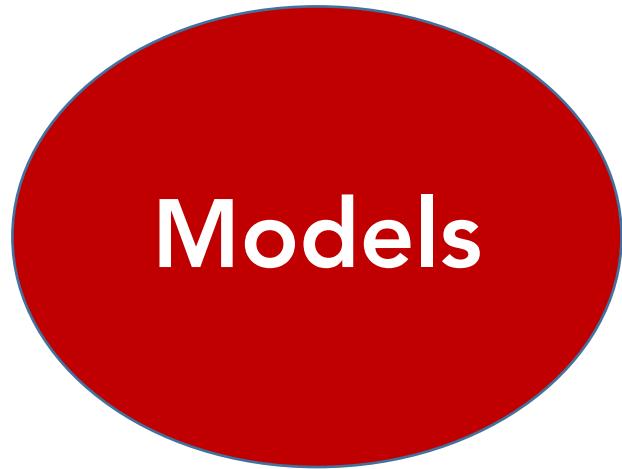
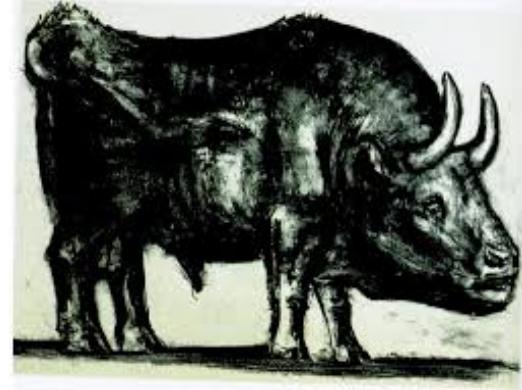
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<http://trivediresearch.org.ohio-state.edu/>





# QUANTUM MATTER

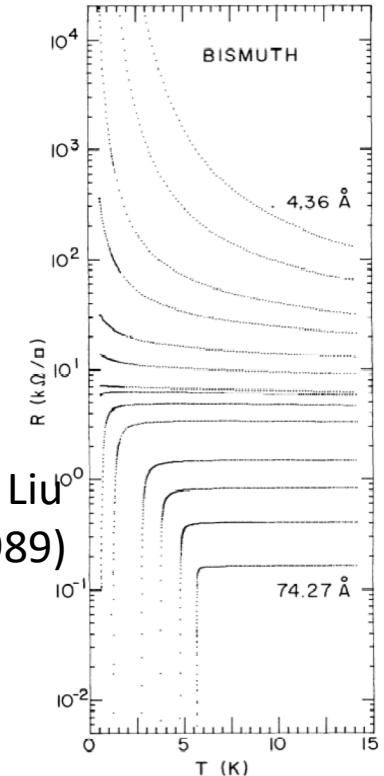


# Superconductor-Insulator Transitions (SIT)

- **Bosonic systems:**

- \* Bose Hubbard model [MPA Fisher et al (1989)]
- \* Mott-SF in optical lattices
- \* Josephson junction arrays

2D films: Haviland, Liu<sup>00</sup>  
& Goldman, PRL (1989)

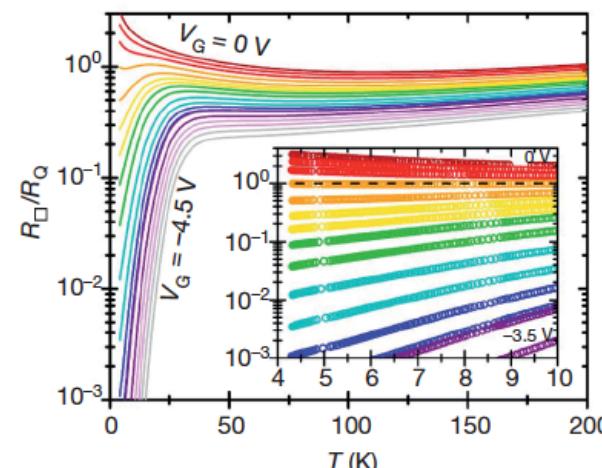


- **Fermionic systems:**

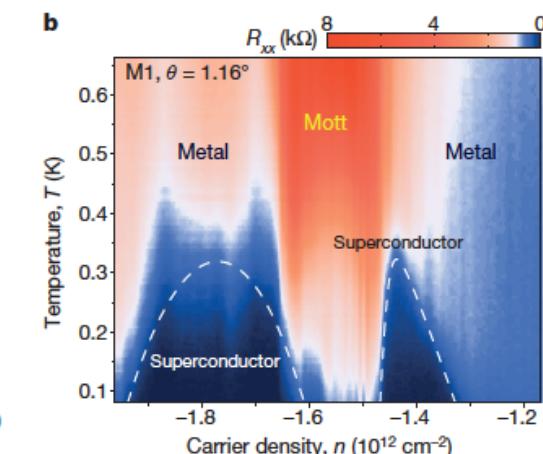
SIT driven by

- \* disorder
- \* magnetic field
- \* doping
- \* gating
- \* pressure

...



LSCO: Bollinger et al,  
Nature (2011)



MA-TBG: Cao et al,  
Nature (2018)

## Local spectroscopies

- One-particle density of states
- Josephson spectroscopy
- Compressibility
- Diamagnetic susceptibility

## Theoretical Methods

- Mean field theory;  
Pairing of exact eigenstates  
Inhomogeneous Bogoliubov MFT
- Quantum Monte Carlo  
(-U Hubbard): no sign problem for any  $\mu$
- QMC for Quantum XY Model ( $n, \Phi$ )
- Maximum entropy to extract spectral functions

# Insulator to Superconductor Transition

## Seeds of pairing found in the insulator

Band insulator to SC:

Loh, Randeria, Trivedi, Chen, Scalettar, Phys. Rev. X 6, 021029 (2016)

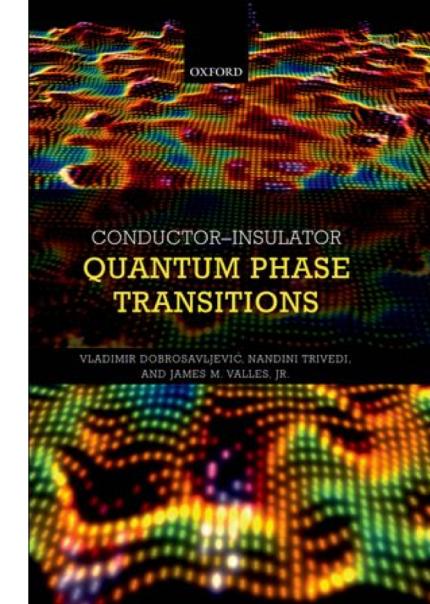
Disorder driven SC to Insulator:

Ghosal, Randeria, Trivedi, PRB 65 014501 (2001); PRL 81 3940 (1998)

Bouadim, Loh, Randeria, Trivedi, Nat. Phys. 7, 884 (2011)

Magnetic field driven SC to insulator:

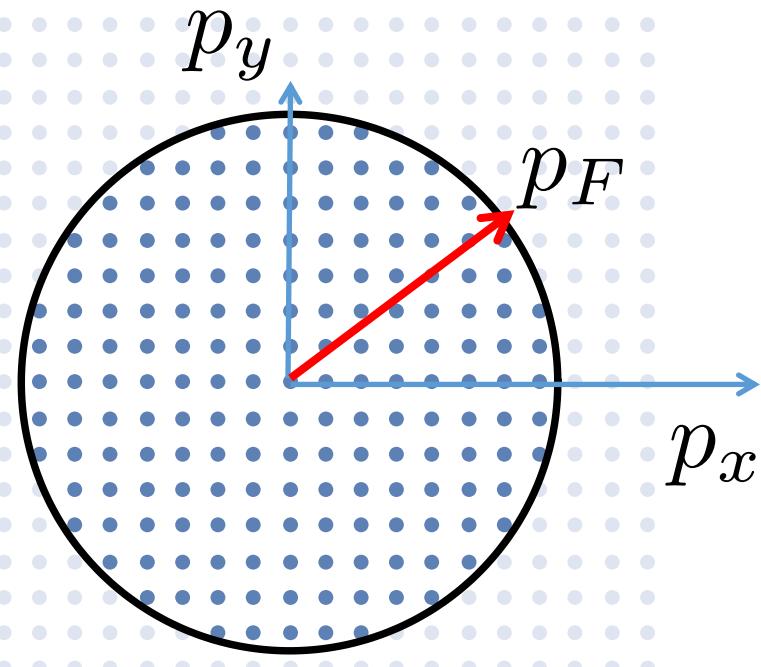
Datta, Banerjee, Trivedi, Ghosal, preprint



Conductor-Insulator Quantum Phase Transition (ed. V. Dobrosavljevic, N. Trivedi, and J. M. Valles) Oxford (2012)

## Breakdown of the standard BCS paradigm

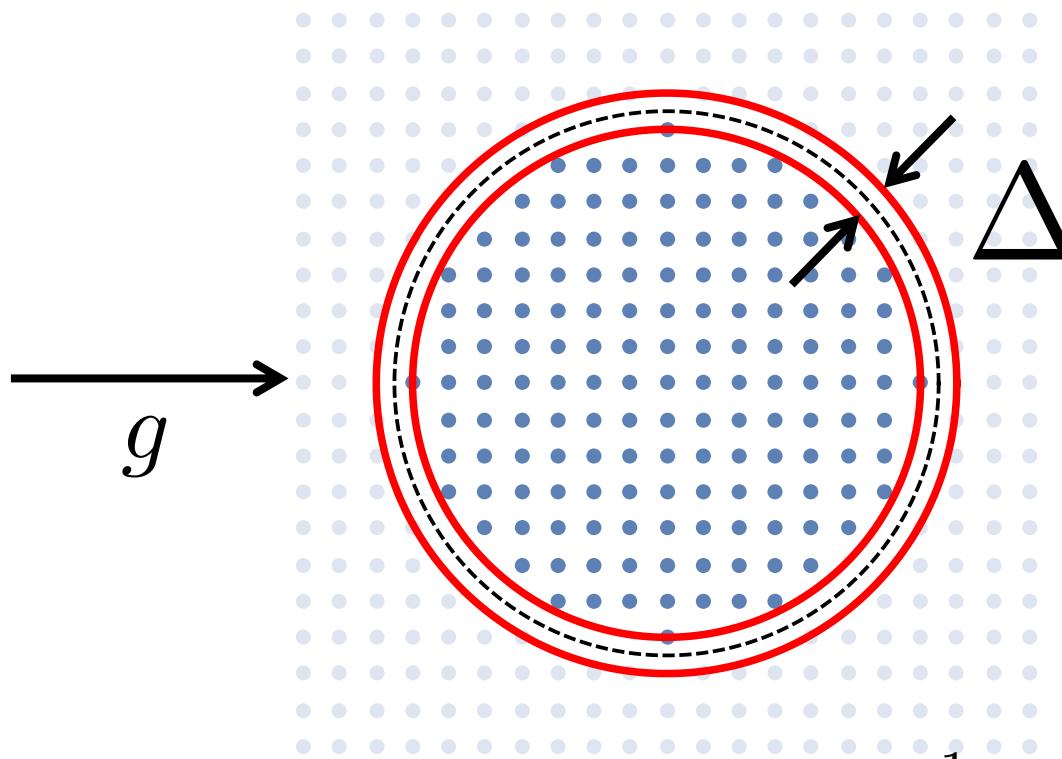
# BCS Paradigm



$$p = h/\lambda \rightarrow \text{wave length of electron wave}$$

$$E_F = \frac{p_F^2}{2m} \quad \text{Fermi energy}$$

# Instability of Fermi surface



$$\Delta \sim E_m e^{-\frac{1}{g}}$$

$$\Delta < E_m < E_F$$

Binding  
energy  
of pair

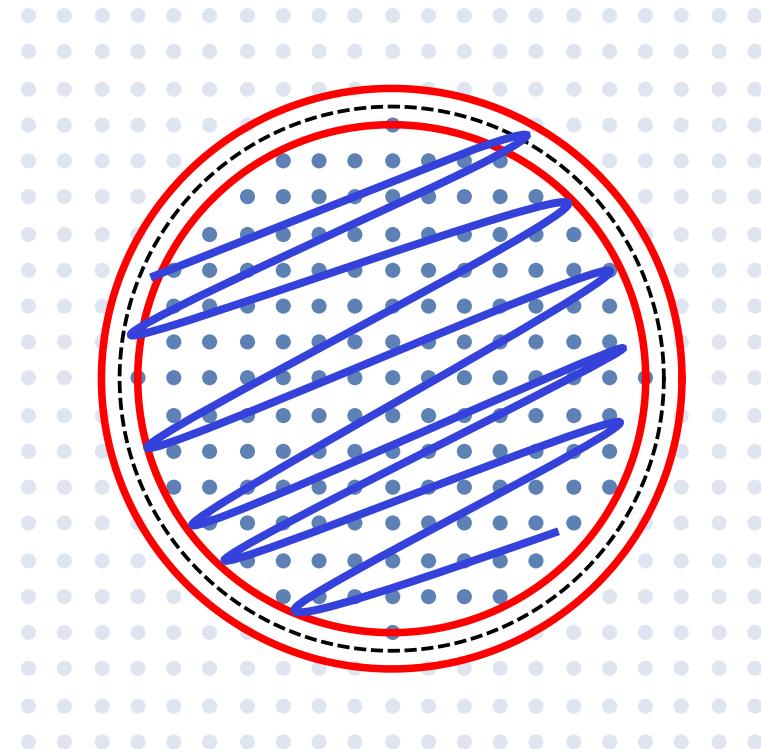
# How can the BCS paradigm break down?

- (1) Strong coupling to glue:  
Fermi sphere greatly perturbed

$$T_c \not\propto \Delta$$

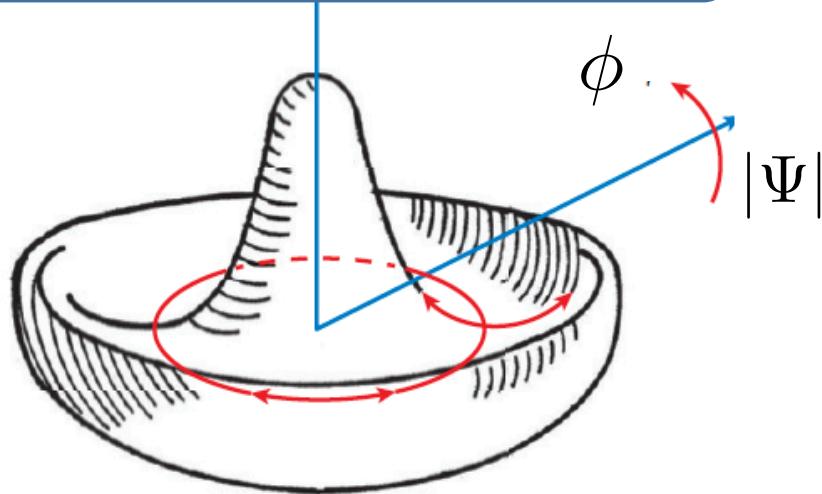
- (2) Non-adiabatic limit:  
electrons are *slower* than the mode

$$\Delta \not\propto E_m e^{-\frac{1}{g}}$$



# Spontaneously broken continuous symmetry

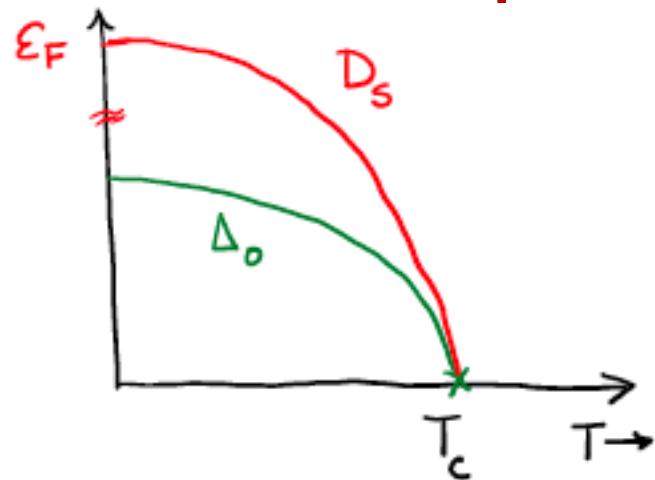
$$\langle a \rangle \equiv \Psi = |\Psi| e^{i\phi}$$



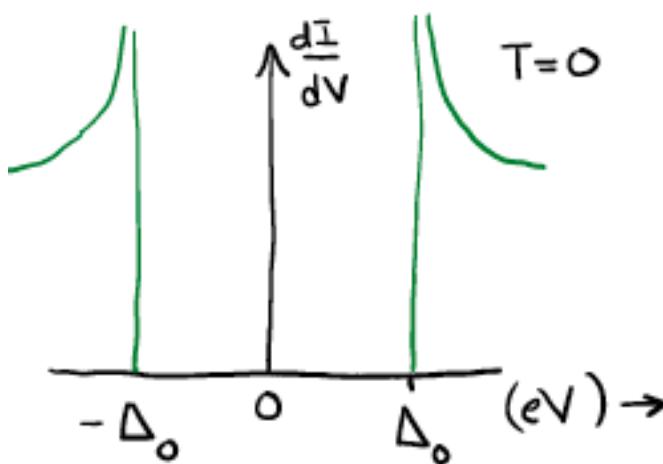
Gapless azimuthal phase mode: Goldstone mode

Gapped radial amplitude mode: Higgs mode

# Superconductor: gap and stiffness



Single particle gap  
~ Order Parameter  
~ Pairing amplitude



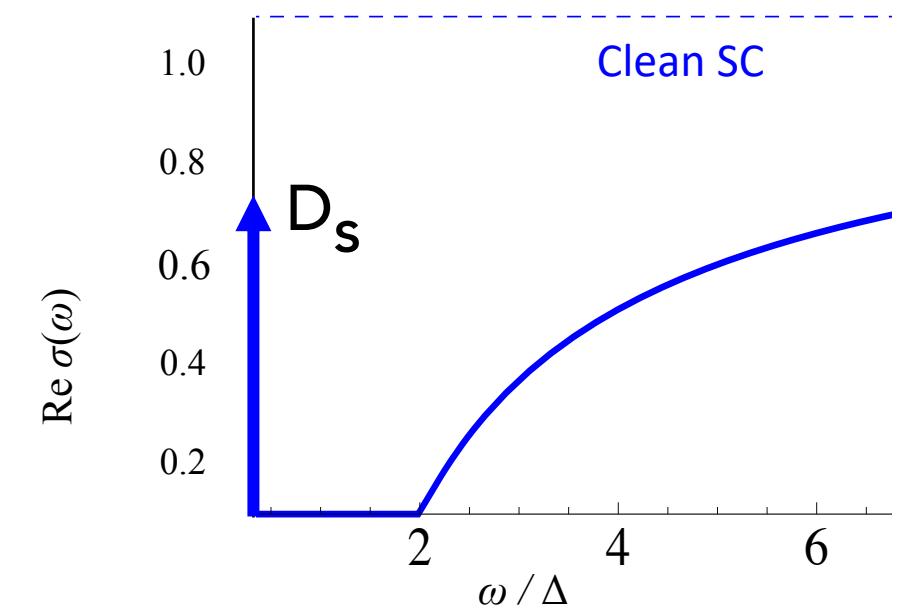
Superfluid stiffness  
~ Superfluid density

$$\int_0^\infty \text{Re } \sigma(\omega) d\omega = \#$$

$$D_s + \int_{0^+}^\infty \text{Re } \sigma(\omega) d\omega = \#$$

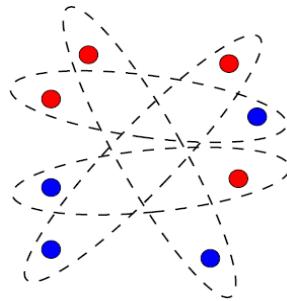
GS: ↑↑↑↑↑↑  
External twist ↑ →

$$D_s = \frac{\partial^2 f}{\partial \theta^2}$$



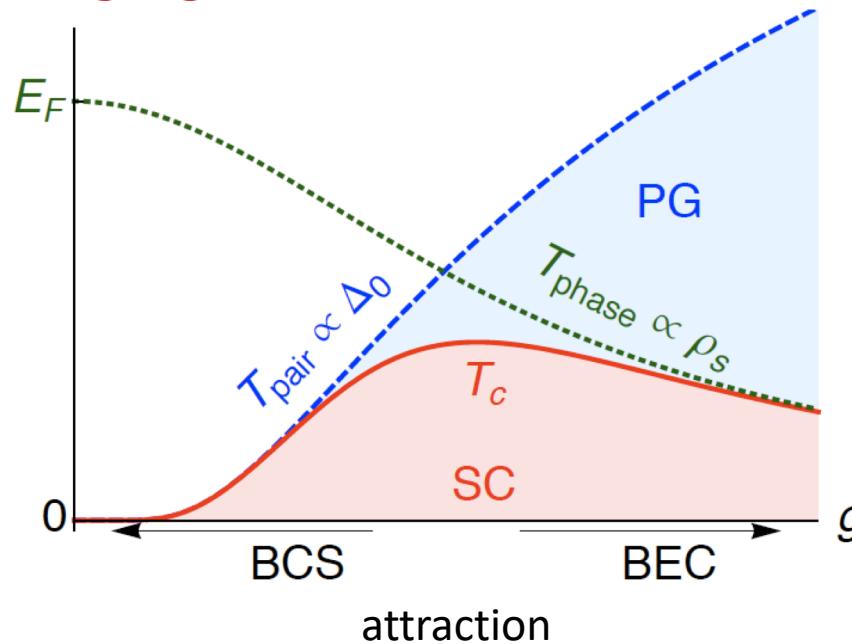
# Strong glue: BCS-BEC Crossover

weak attraction:  
pairing and coherence  
occur at the same  
temperature



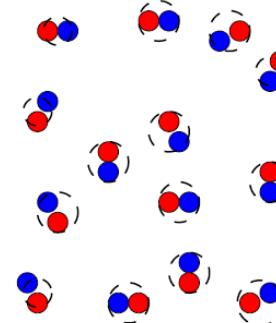
## BCS limit

- cooperative Cooper pairing
- pair size  $\gg k_F^{-1}$



$$T_c = \min(\Delta_0, \rho_s)$$

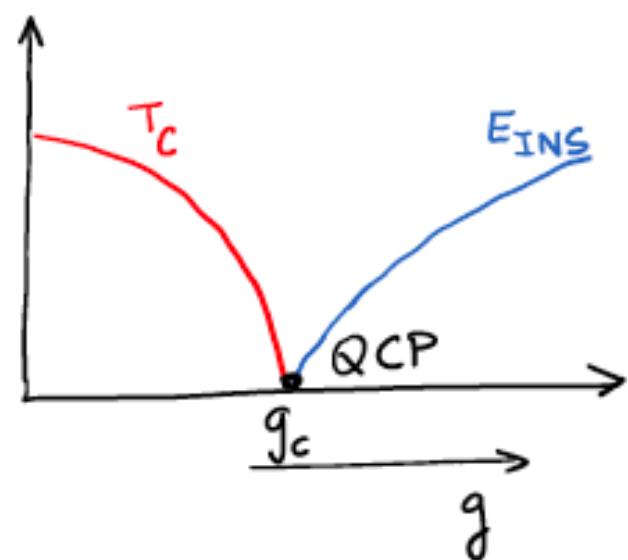
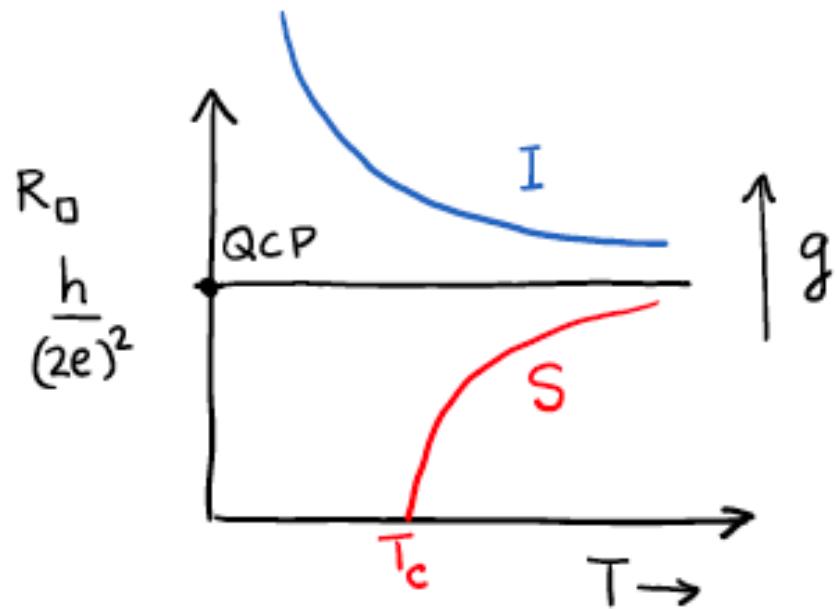
strong attraction:  
pairing and coherence  
occur at **different**  
temperatures



## BEC limit

- tightly bound molecules
- pair size  $\ll k_F^{-1}$

# Superconductor – Insulator Transition



**Big open questions:**

- What type of insulator? Nature of SC?
- What is the energy scale that goes soft at the transition from the insulating side?
- QCP: fermionic or bosonic?
- Is there a Bose metal phase around the transition?

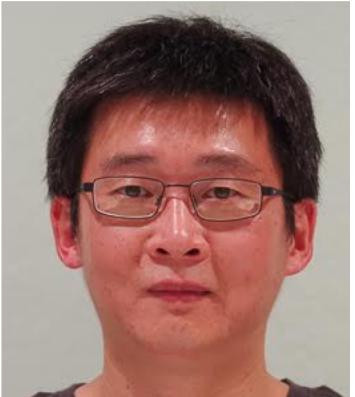
# Part I

# Insulator to Superconductor Transition

## Seeds of pairing found in the insulator

Band insulator to SC:

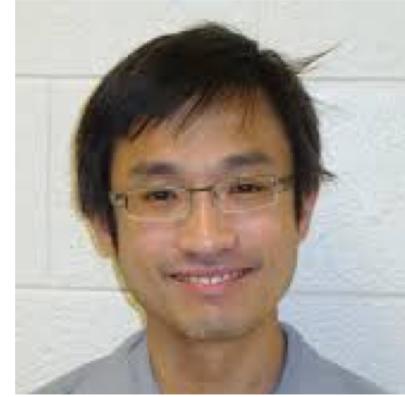
Loh, Randeria, Trivedi, Chen, Scalettar, Phys. Rev. X 6, 021029 (2016)



Chia-Chen Chang



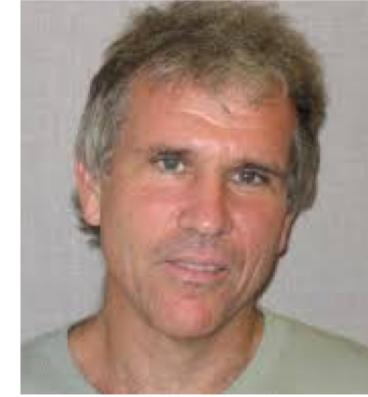
Tamaghna Hazra



Yen Lee Loh



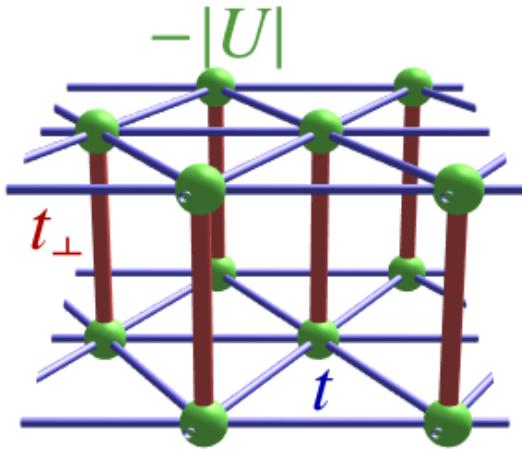
Mohit Randeria



Richard Scalettar

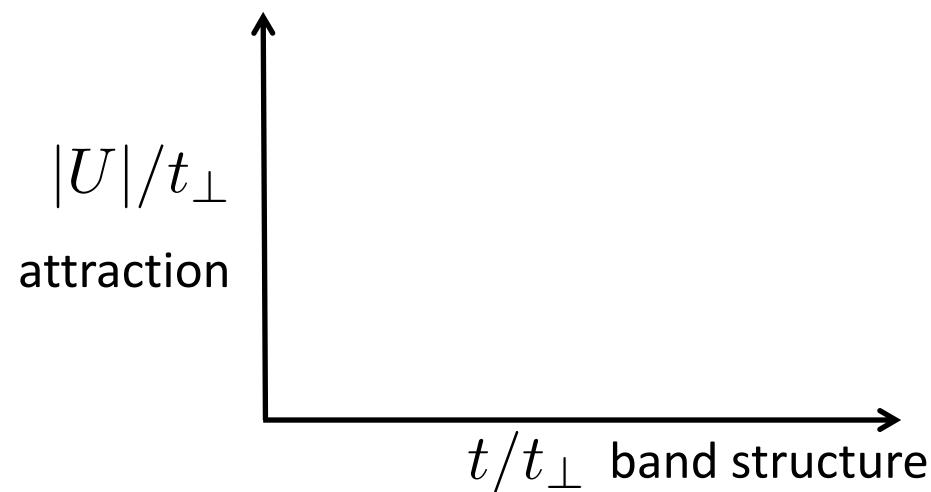
Can an insulator become a SC? [*at same doping*]  
How does SC arise when there is *no* Fermi surface?

# Triangular bilayer attractive Hubbard model



$$H = -t \sum_{\langle ij \rangle_{\parallel} \sigma} (c_{i\sigma}^\dagger c_{j\sigma} + h.c.)$$

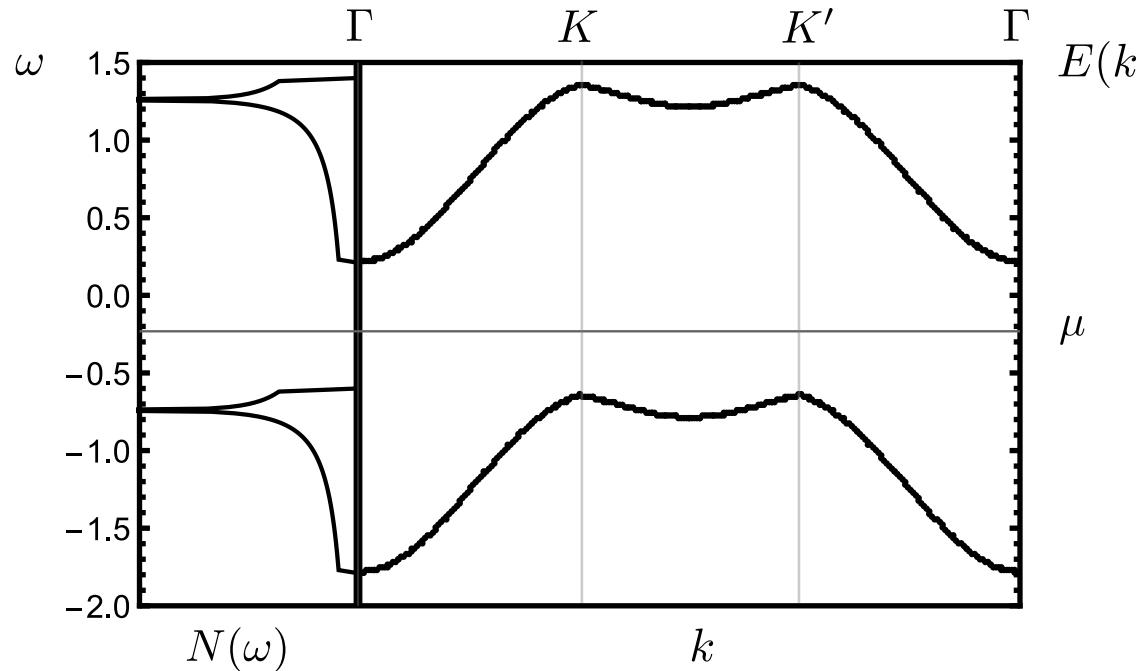
$$- t_{\perp} \sum_{\langle ij \rangle_{\perp} \sigma} (c_{i\sigma}^\dagger c_{j\sigma} + h.c.)$$



$$- |U| \sum_i (n_{i\uparrow} - 1/2)(n_{i\downarrow} - 1/2) - \mu \sum_{i\sigma} n_{i\sigma}$$

Fix  $T=0$ ,  $t_{\perp} = 1$   $\langle n \rangle = 1$

# Triangular lattice TB band structure

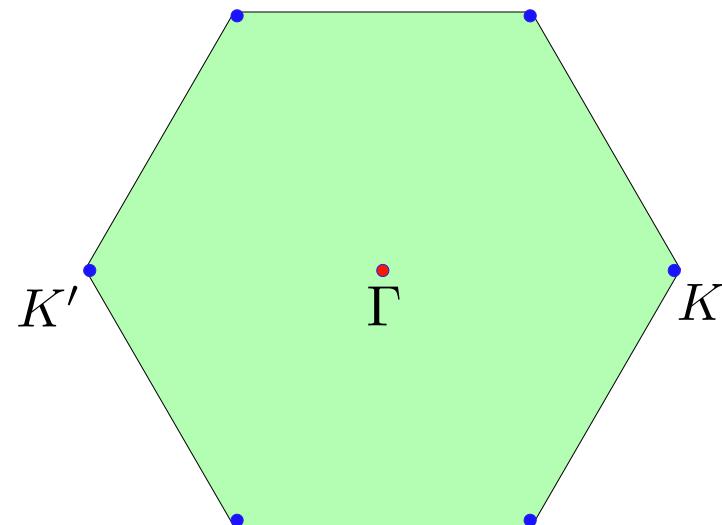


$E(k)$

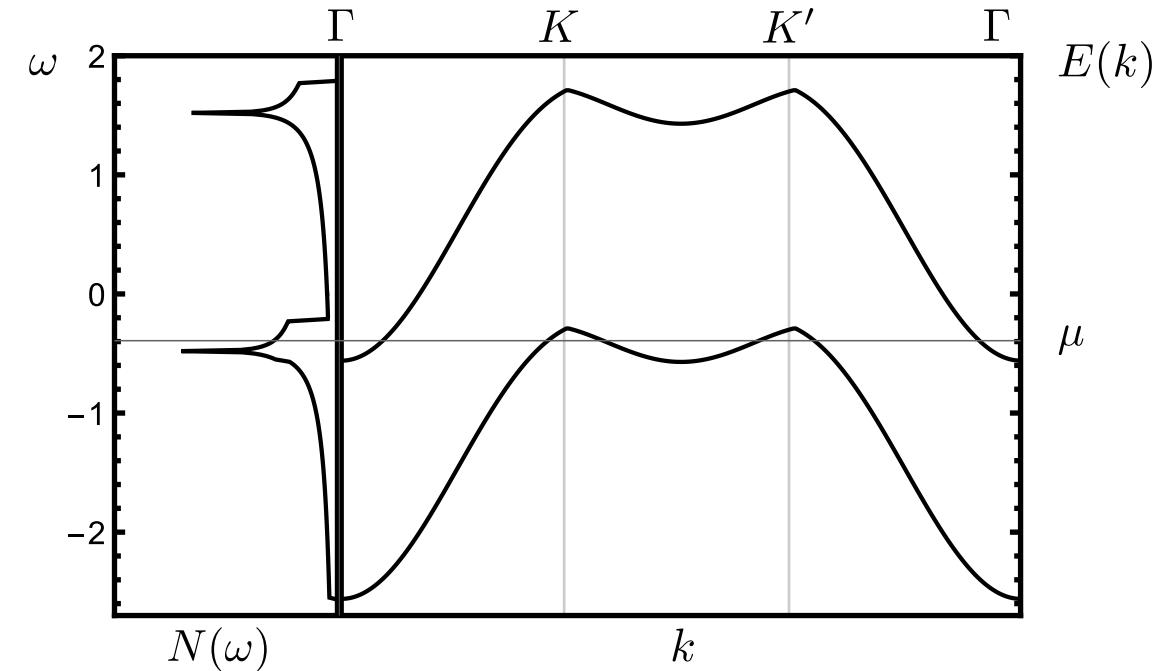
$\mu$

$N(\omega)$

$k$



$t/t_{\perp}$

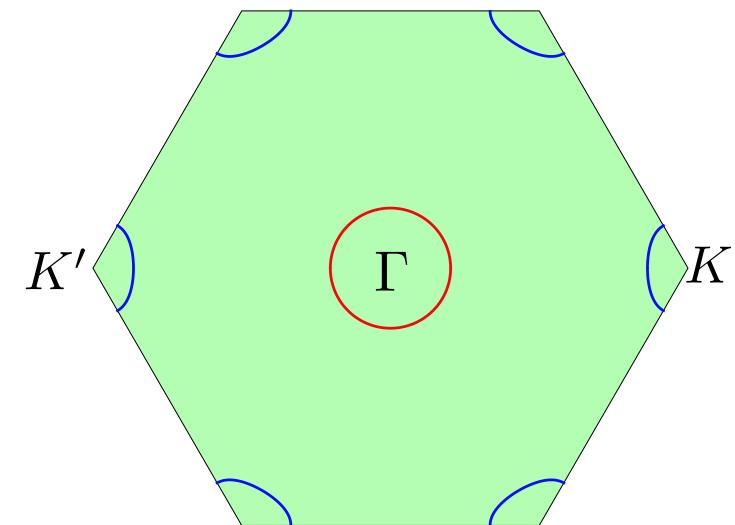


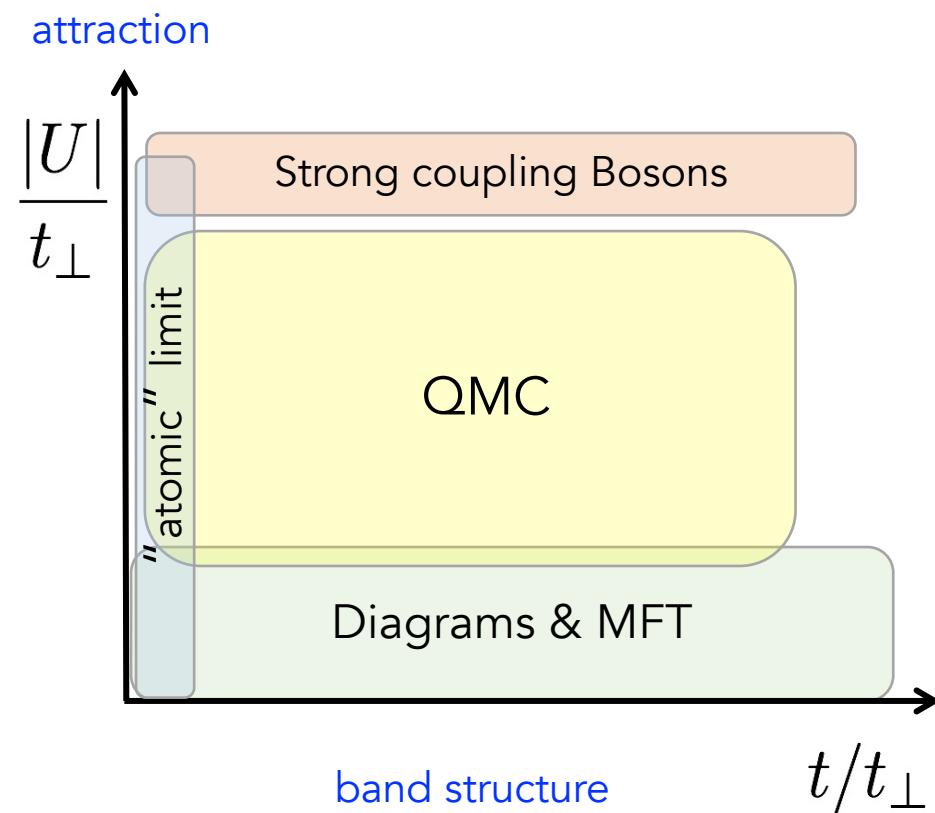
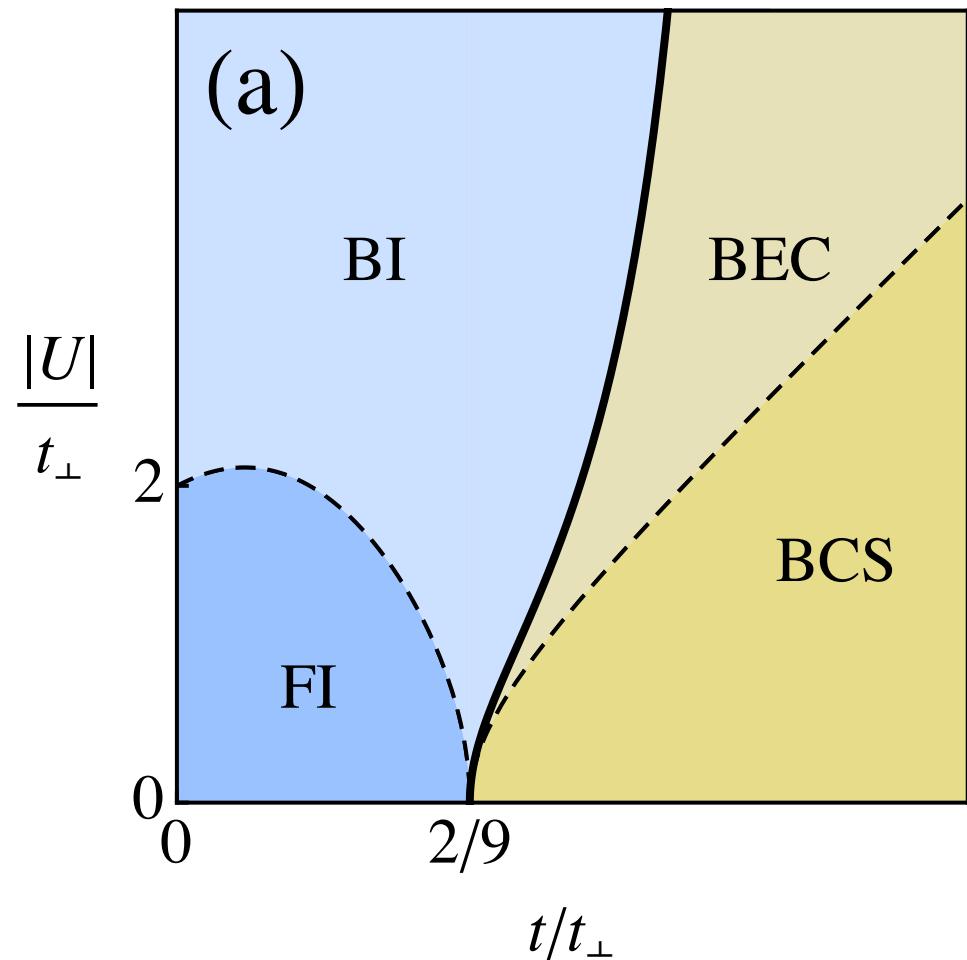
$E(k)$

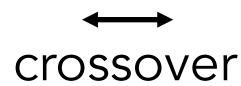
$\mu$

$N(\omega)$

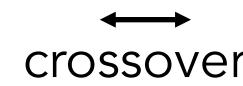
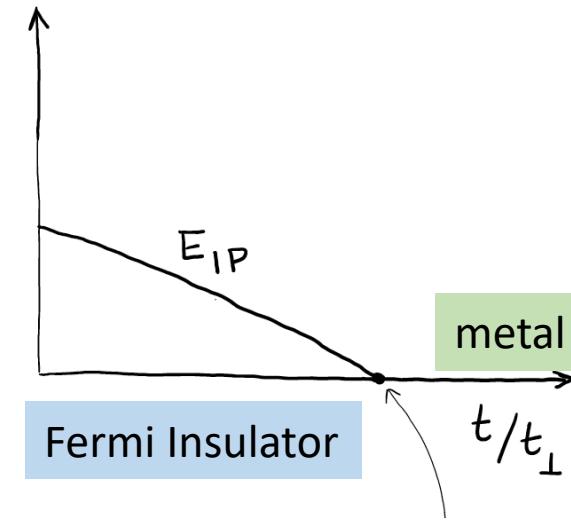
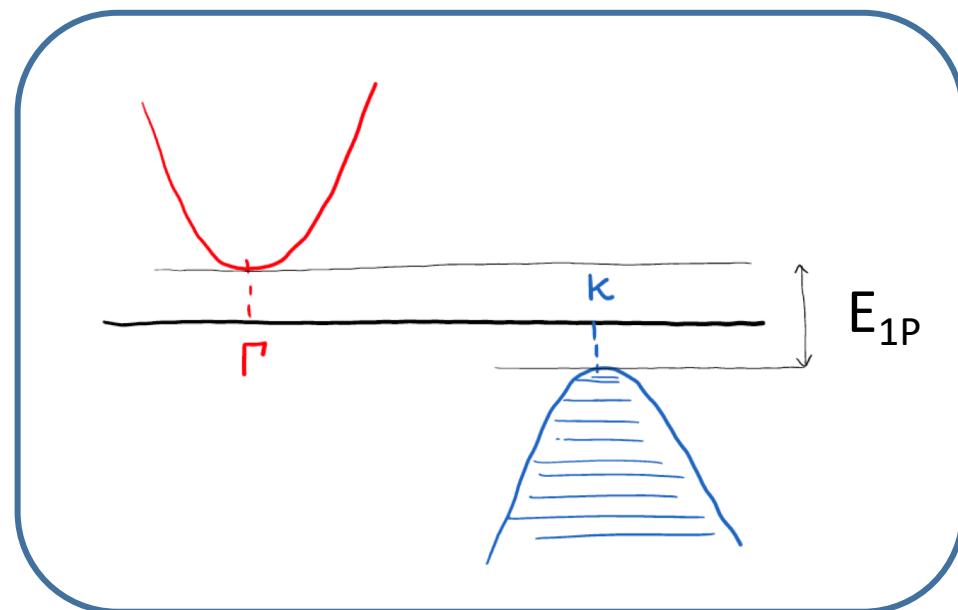
$k$



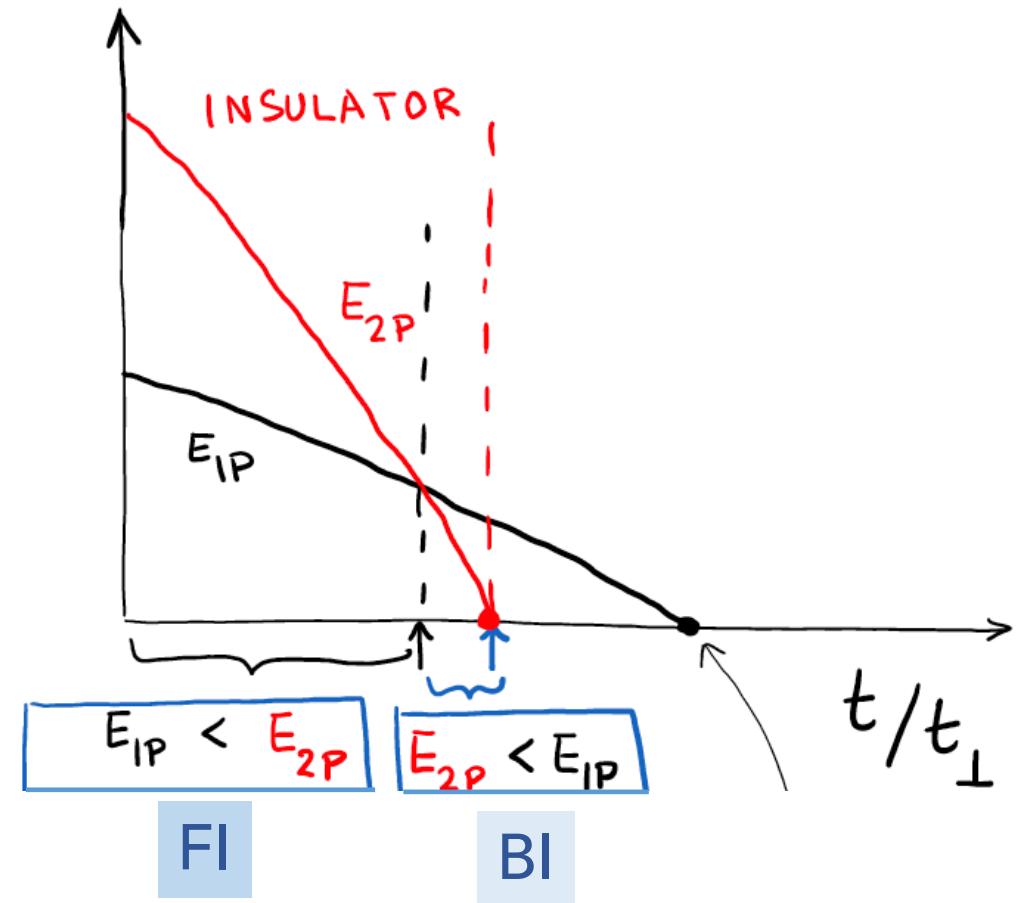
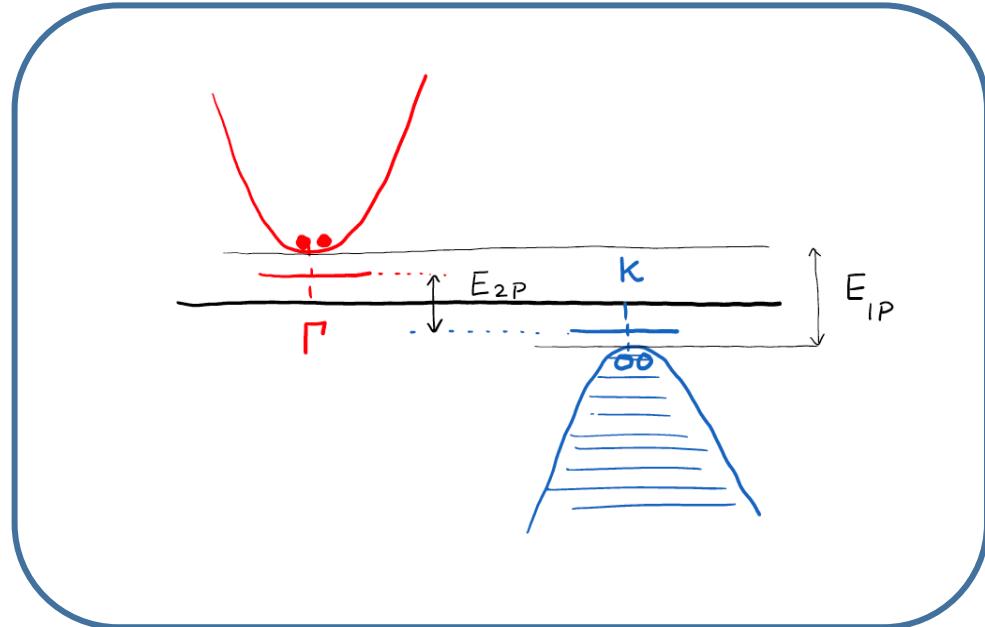


$t/t_{\perp}$   crossover

SC- Insulator  
Transition

 crossover

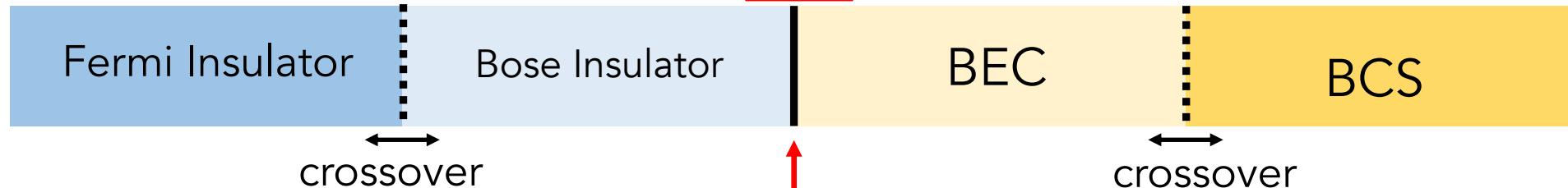
For Fixed  $U$



For Fixed U

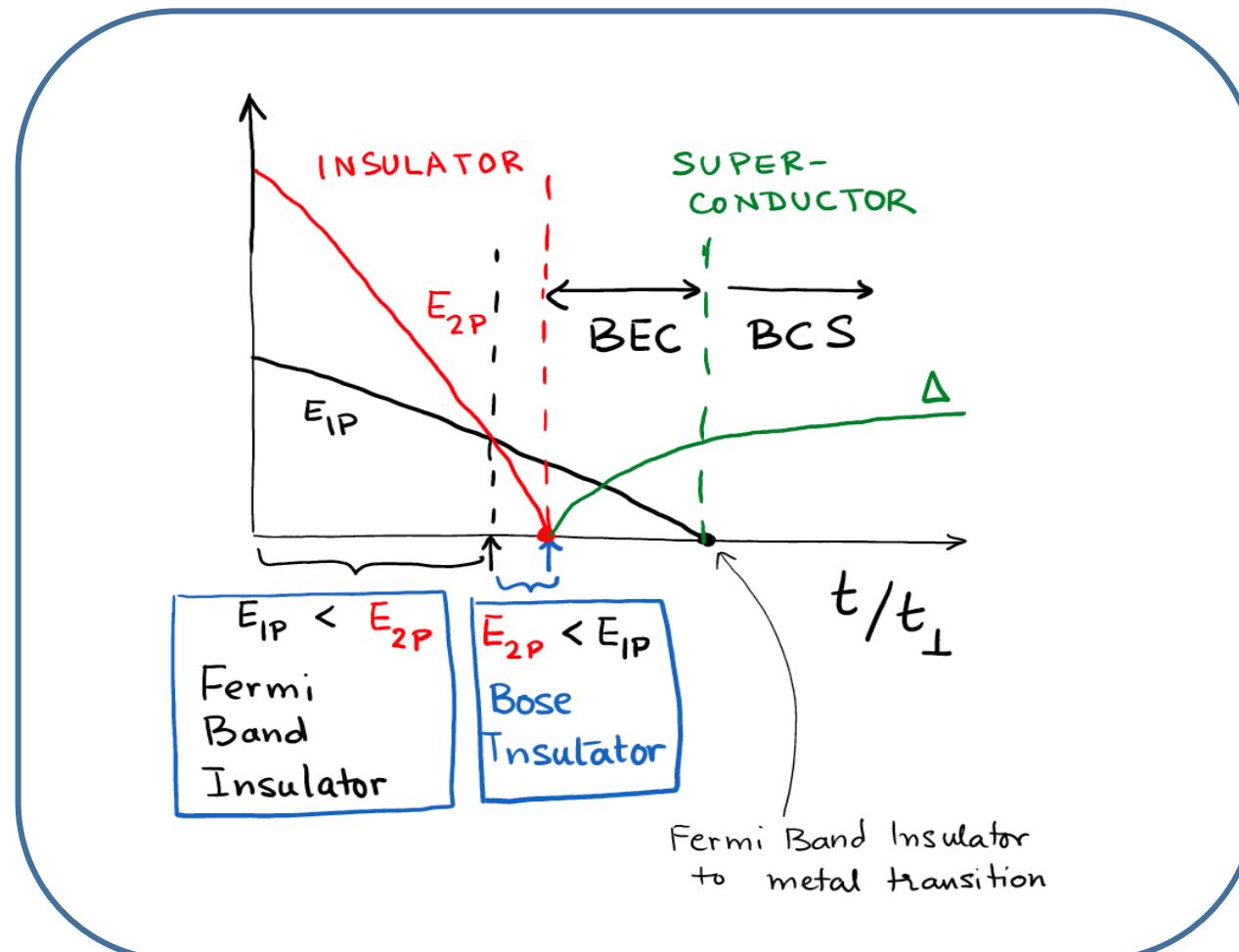
SIT

$t/t_{\perp}$

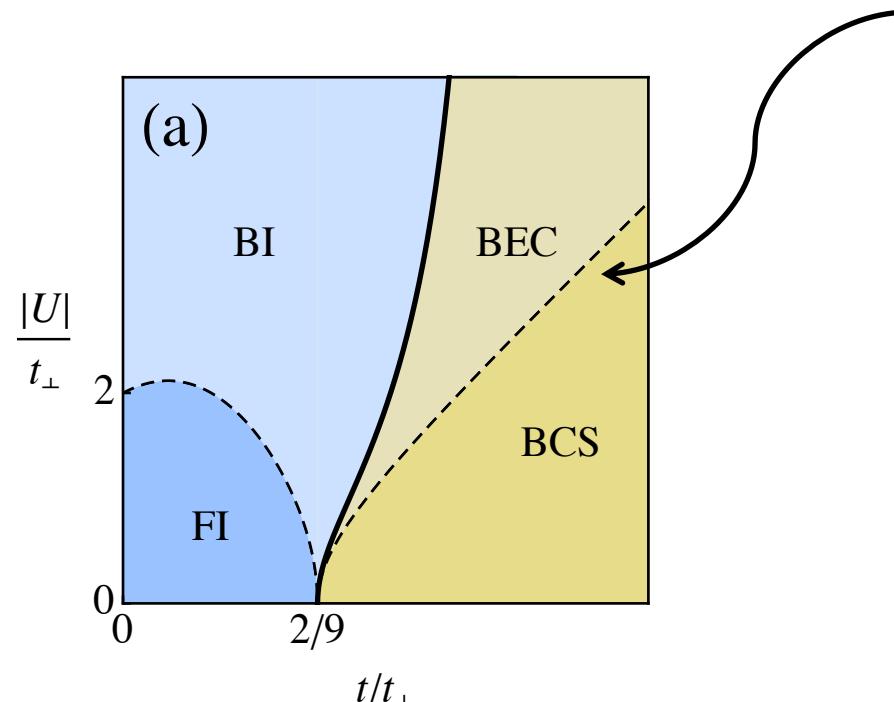


$$E_{SP} = \begin{cases} E_{IP} \\ \sqrt{E_{IP}^2 + \Delta^2} \\ \Delta \end{cases}$$

INS  
BEC  
SC



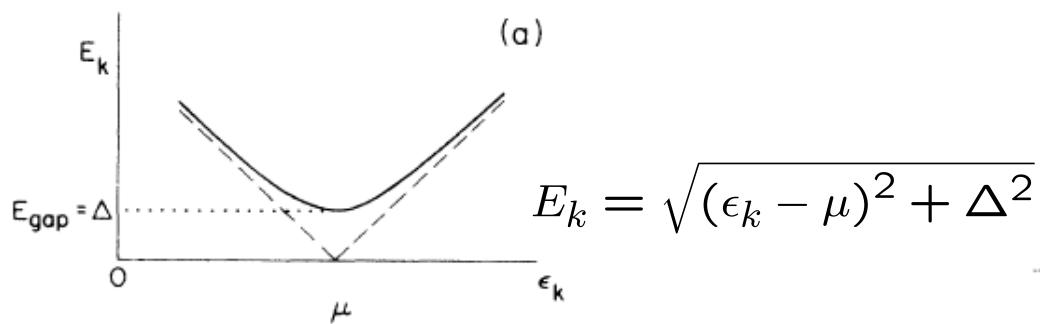
# How to identify the BCS & BEC regimes in the crossover?



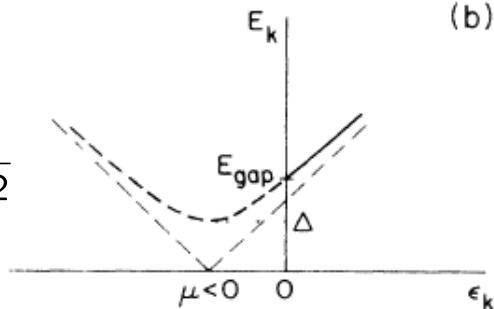
2 Predictions:

- Topology of "Minimum Gap Locus"—  
ARPES (angle resolved photoemission spectroscopy)
- Gap-edge singularity in DOS – tunneling

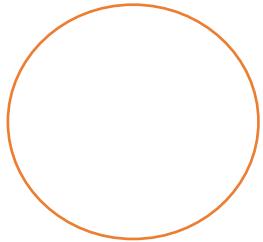
BCS regime ("weak pairing")



BEC regime ("strong pairing")



Minimum gap locus in k-space



contour

$$\epsilon_k = \mu$$

or  $k = "k_F"$



point

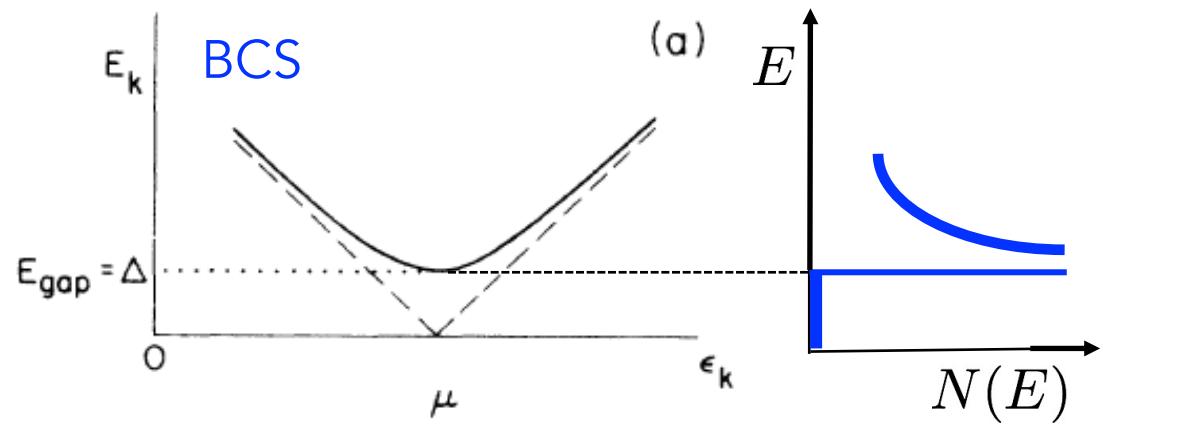
$$\epsilon_k = 0$$

or  $k = 0$

# Crossover from BCS to BEC regime

- \* Topology of "Minimum Gap Locus"
- \* Gap-edge singularity in DOS

$$E_{\text{gap}} \equiv \min_{\epsilon_k \geq 0} [(\epsilon_k - \mu)^2 + |\Delta_{\mathbf{k}}|^2]^{1/2}$$



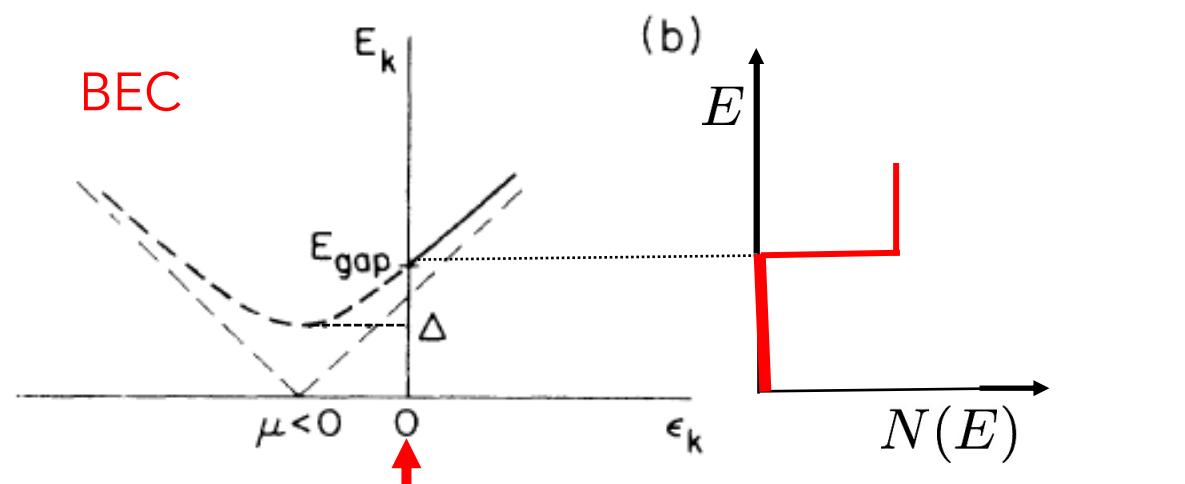
Min gap locus is  
contour  $\epsilon_{\mathbf{k}} = \mu$

BCS Min gap locus  $\leftrightarrow G(\mathbf{k}, \omega=0) = 0$

$1/\text{(square-root)}$   
In DOS

First, simple case of single band

$$E_{\text{gap}} = \begin{cases} \Delta & \text{for } \mu > 0, \leftarrow \text{BCS} \\ (\mu^2 + \Delta^2)^{1/2} & \text{for } \mu < 0, \leftarrow \text{BEC} \end{cases}$$



Min gap locus is  
point  $\mathbf{k} = 0$

Sensarma, Randeria & Trivedi PRL 98, 027004 (2007)

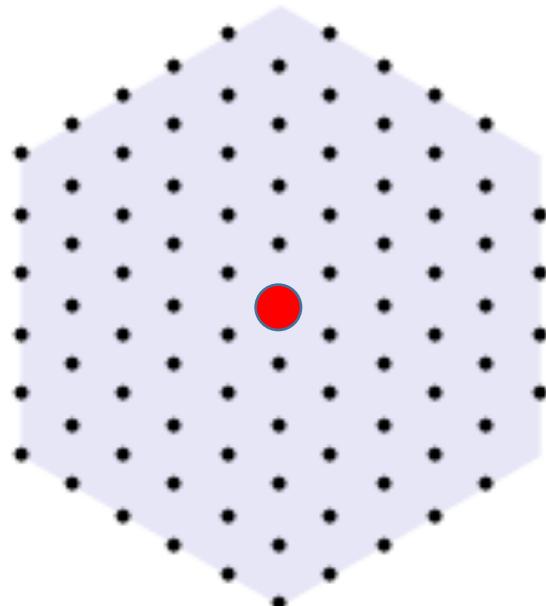
DOS has jump  
discontinuity (2D)



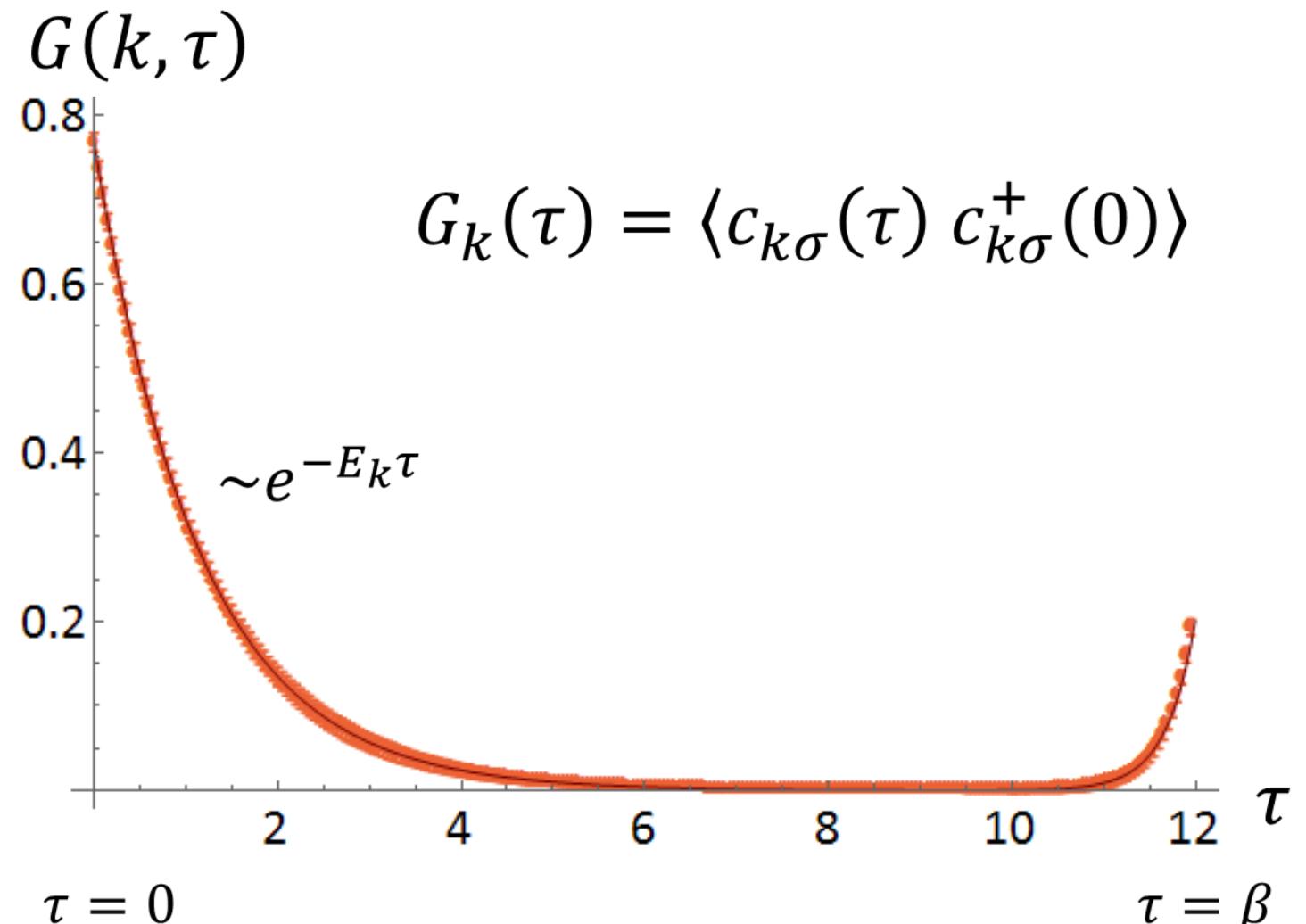
# Single particle gap

QMC Results

Tamaghna Hazra

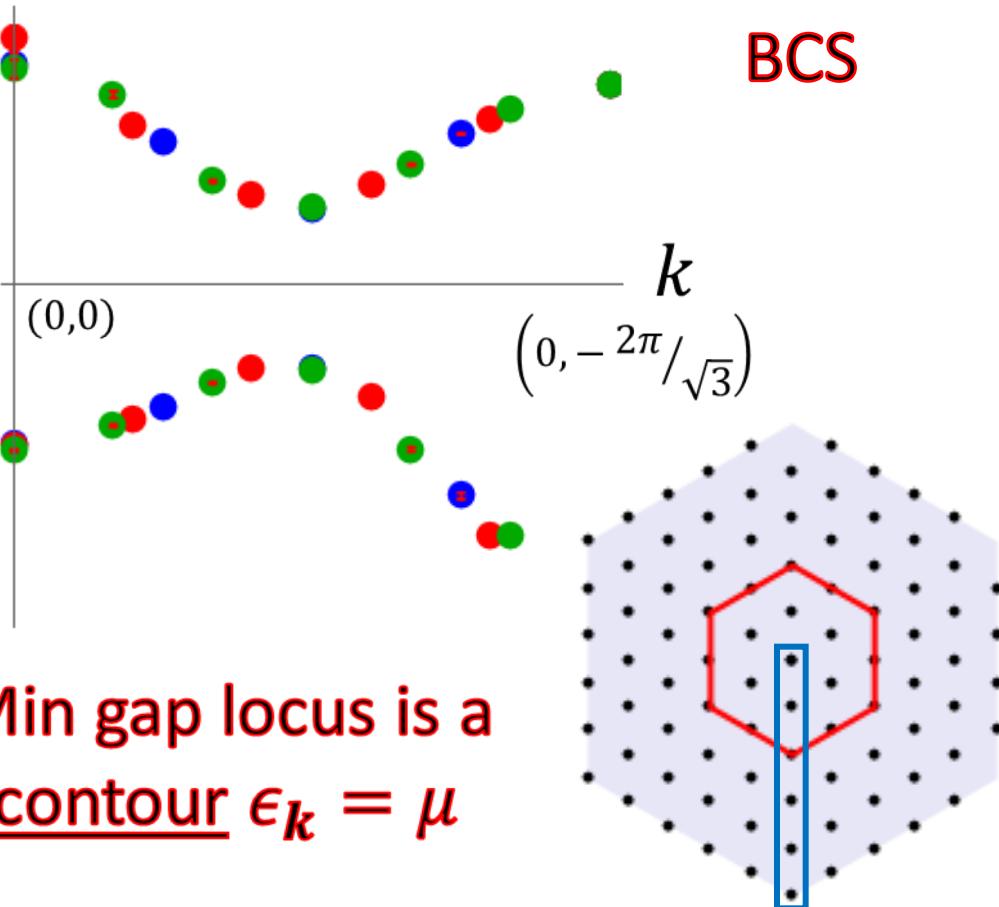


$10 \times 10 \times 2$  lattice  
 $|U| = 4t_{\perp}, t = 0.2t_{\perp}$   
 $k_z = \pi, \beta t_{\perp} = 12, \Delta\tau = 0.05$

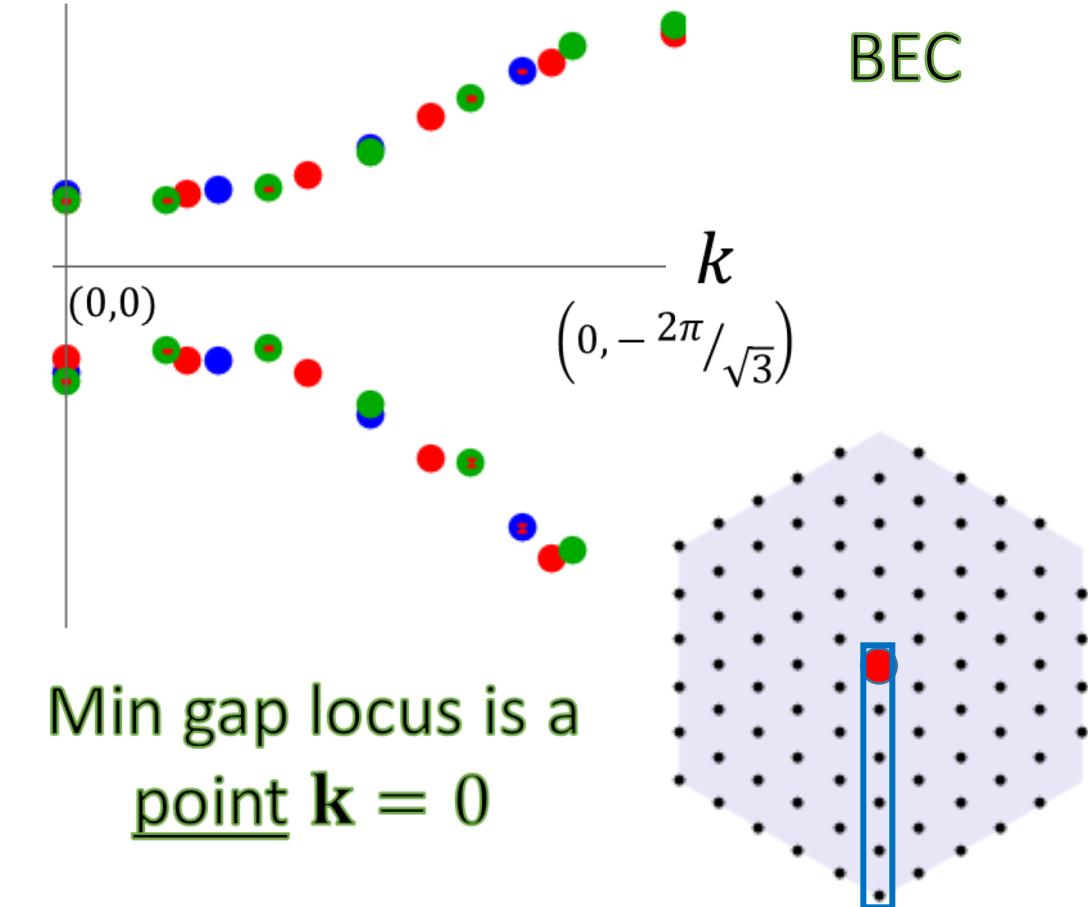


# BCS-BEC Crossover : Topology of Minimum gap locus

$E(k)$      $t = 0.5t_{\perp}$



$E(k)$      $t = 0.37t_{\perp}$



QMC Results

●  $12 \times 12 \times 2$  ●  $10 \times 10 \times 2$  ●  $8 \times 8 \times 2$

$$\frac{\Delta}{\epsilon_F} =$$

$\text{Fe}_{1+y}\text{Se}_x\text{Te}_{1-x}$

Tuned by changing  
the Fermi energy  
controlled by tuning  $y$

- Fermi energy estimated from the unoccupied bandwidth for the hole pocket
- SC gap obtained from the energy of the coherence peaks

0.16

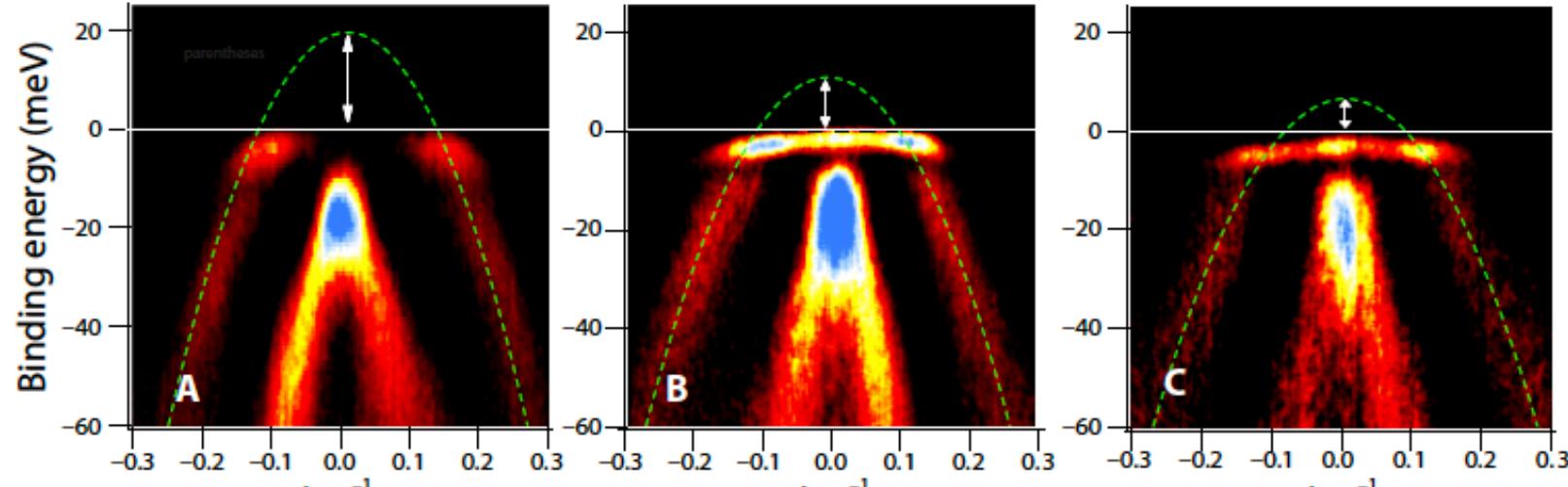
0.3

0.5

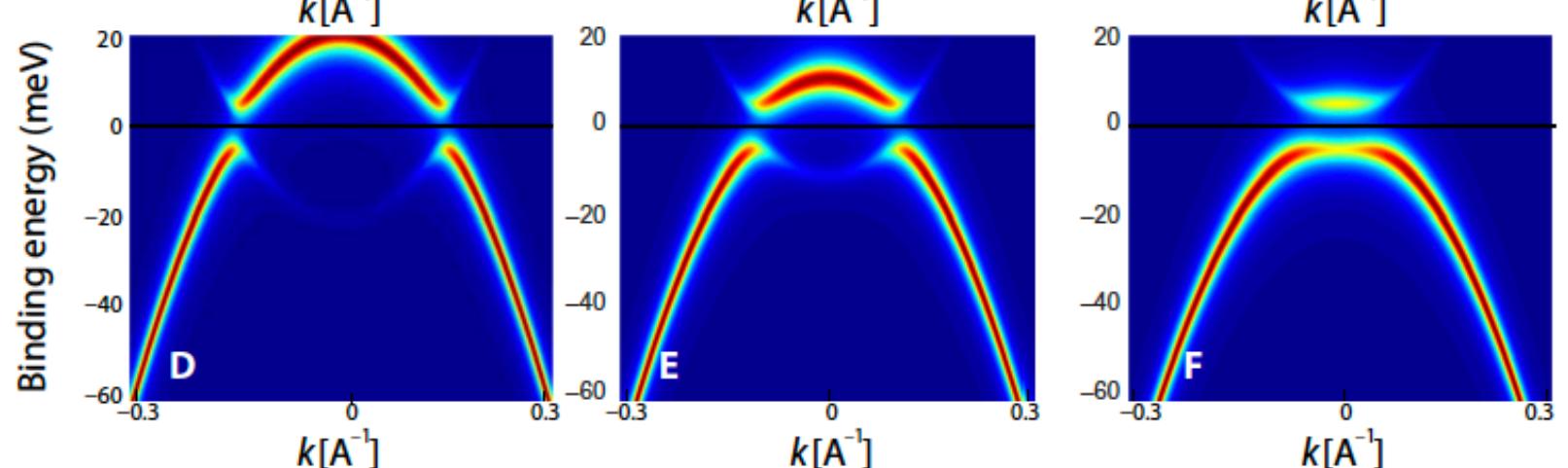
Decreasing  $y$  in  $\text{Fe}_{1+y}\text{Se}_x\text{Te}_{1-x}$

BCS

BEC



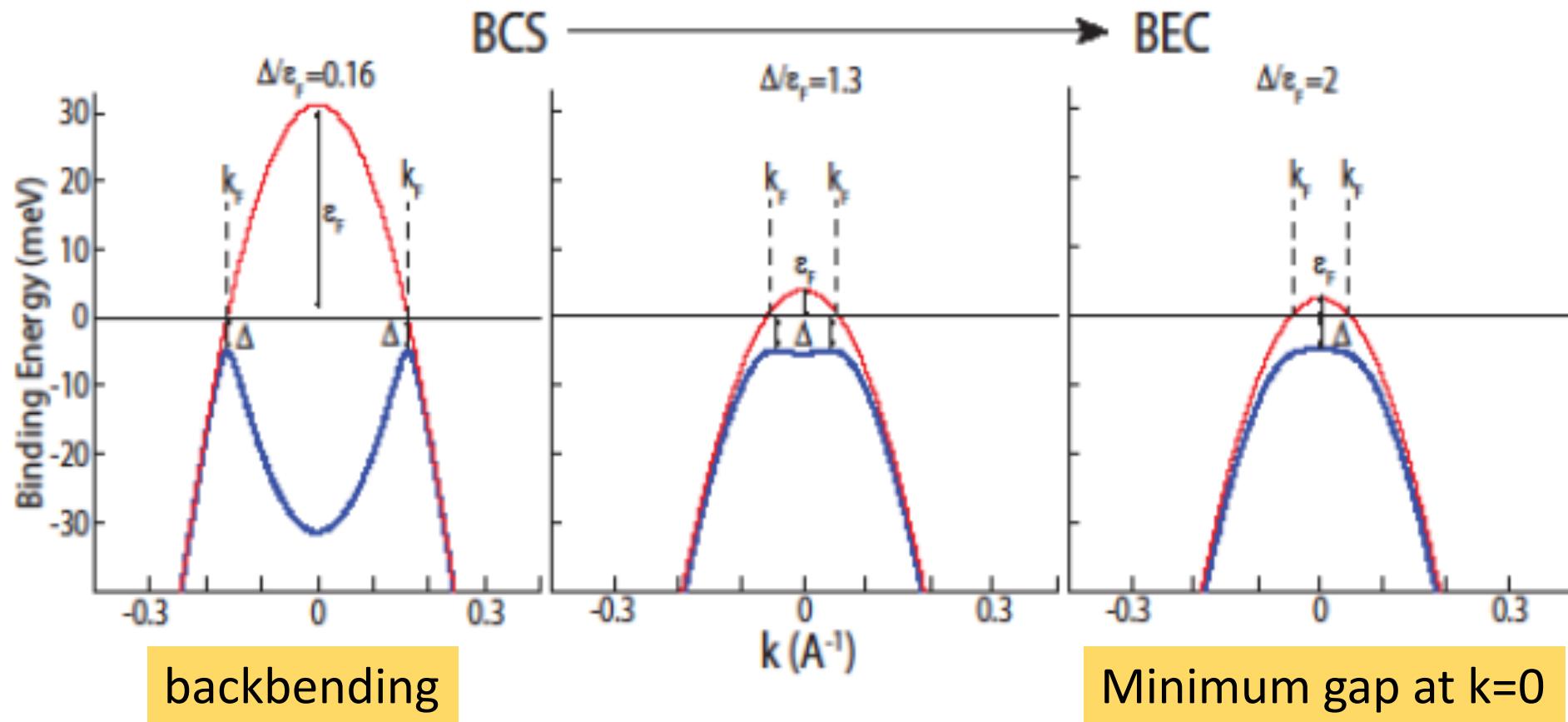
Expt



Theory

backbending

Minimum gap at  $k=0$



# Insulator-Superconductor Transition:

