Part II

Insulator to Superconductor Transition

Seeds of pairing found in the insulator

Band insulator to SC:
Loh, Randeria, Trivedi, Chen, Scalettar, Phys. Rev. X 6, 021029 (2016)

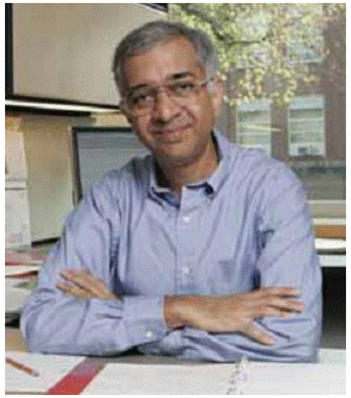
Disorder driven SC to Insulator:
Ghosal, Randeria, Trivedi, PRB 65 014501 (2001); PRL 81 3940 (1998)
Bouadim, Loh, Randeria, Trivedi, Nat. Phys. 7, 884 (2011)

Magnetic field driven SC to insulator:
Datta, Banerjee, Trivedi, Ghosal, preprint

Amit Ghosal

Karim Bouadim

Yen Lee Loh



Mohit Randeria

Disorder driven Superconductor-Insulator Transition SIT

Anderson Localization

?

BCS s-Superconductivity

Breakdown of both paradigms!

SC + Disorder

Weak disorder: $k_F \ell \gg 1$

No effect on thermodynamics: Anderson; Abrikosov & Gorkov

Moderate disorder: $k_F \ell \gtrsim 1$

Renormalization of μ^* → Tc suppression
Finkelstein; Fukuyama, Maekawa & Ebisawa

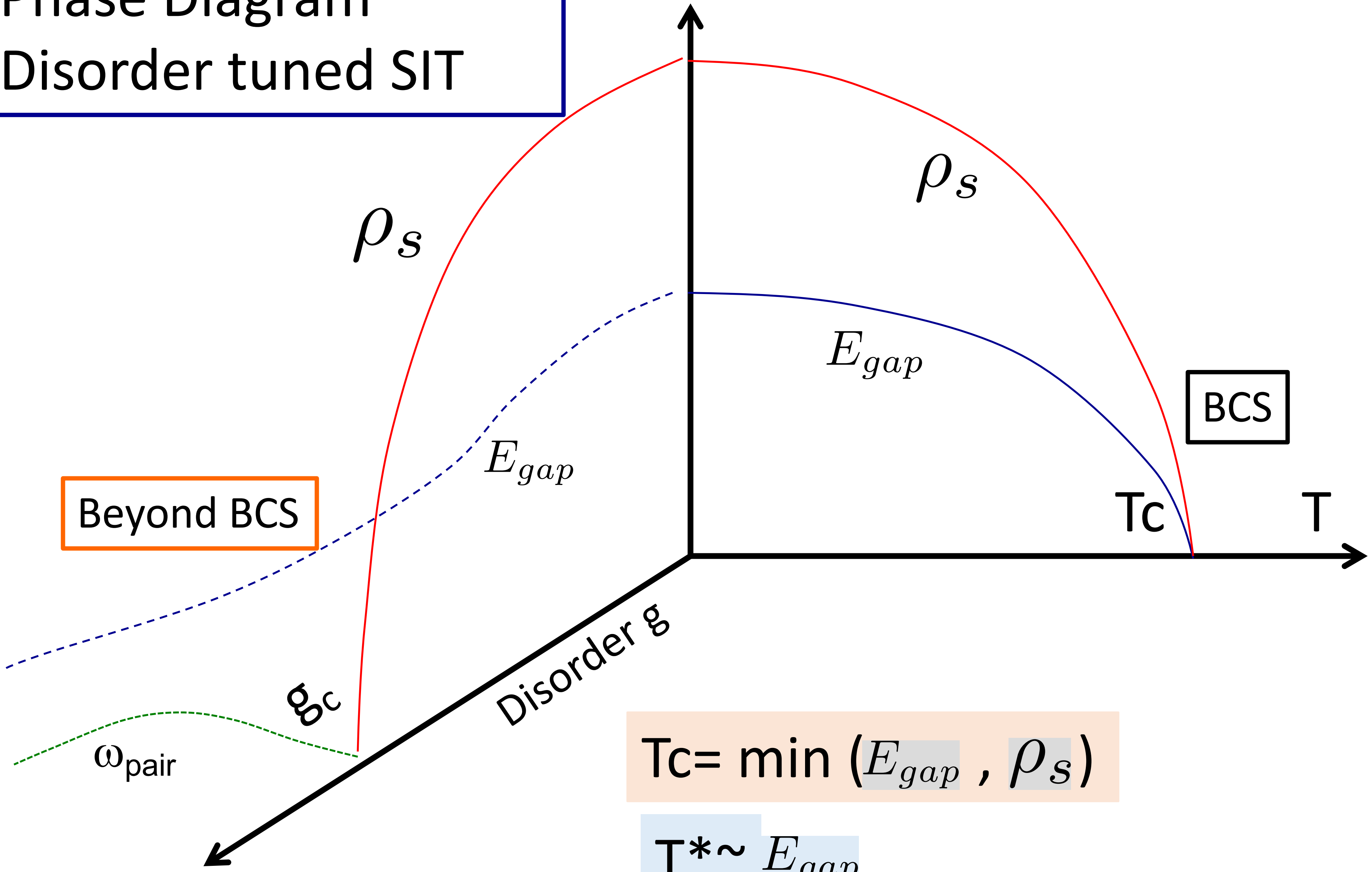
Strong disorder: $k_F \ell \sim 1$

Anderson localization v/s Anderson's theorem
Ma & Lee → Localized superconductors
Kotliar & Kapitulnik

Quantum Critical Point:

Dirty Bosons; MPA Fisher et al.

Phase Diagram
Disorder tuned SIT



Tc= min (E_{gap} , ρ_s)

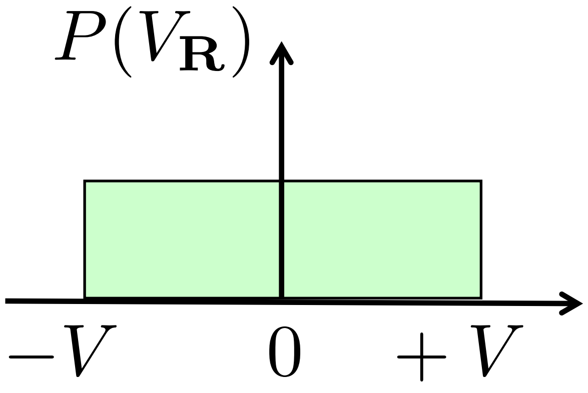
T*~ E_{gap}

Model Hamiltonian

$$H = -t \sum_{\langle \mathbf{R}\mathbf{R}' \rangle \sigma} (c^\dagger_{\mathbf{R}\sigma} c_{\mathbf{R}'\sigma} + c^\dagger_{\mathbf{R}'\sigma} c_{\mathbf{R}\sigma}) - \sum_{\mathbf{R}\sigma} (\mu - V_{\mathbf{R}}) n_{\mathbf{R}\sigma} - |U| \sum_{\mathbf{R}} n_{\mathbf{R}\uparrow} n_{\mathbf{R}\downarrow}$$

Random potential
→ Localization

Attractive interaction
→ Superconductivity



$V = 0$ → Clean S-wave SC

Small |U|: BCS

Large |U|: BEC

$|U| = 0$ → Anderson localization

Ignore: repulsive Coulomb interactions
& random variations in attraction

Theoretical Approaches I:

- **Pairing of Exact Eigenstates:**

  generalize Anderson's approach to inhomogeneous systems

- Small U ✓
- Large systems ✓
- Inhomogeneous ✓
- MFT; No phase fluctuations ×

$$\Delta_\alpha = |U| \sum_\beta M_{\alpha,\beta} \frac{\Delta_\beta}{2E_\beta}, \qquad M_{\alpha,\beta} = \sum_{\mathbf{r}_i} |\phi_\alpha(\mathbf{r}_i)|^2 |\phi_\beta(\mathbf{r}_i)|^2$$

- **Bogoliubov-de Gennes (BdG):**

  locally self-consistent MFT for inhomogeneous systems

- Improved MFT ✓
- No phase fluctuations ×

$$\begin{pmatrix} \hat{K} & \hat{\Delta} \\ \hat{\Delta}^* & -\hat{K}^* \end{pmatrix} \begin{pmatrix} u_n(\mathbf{r}_i) \\ v_n(\mathbf{r}_i) \end{pmatrix} = E_n \begin{pmatrix} u_n(\mathbf{r}_i) \\ v_n(\mathbf{r}_i) \end{pmatrix},$$

$$\Delta(\mathbf{r}_i) = |U| \sum_n u_n(\mathbf{r}_i) v_n^*(\mathbf{r}_i),$$

$$\langle n_i \rangle = 2 \sum_n |v_n(\mathbf{r}_i)|^2.$$

- **Phase Fluctuations:** self-consistent harmonic approximation

Theoretical Approaches II:

○ Determinantal Quantum Monte Carlo (QMC):

-- exact; no sign problem for neg U ✓
-- spatially inhomogeneous density and amplitude ✓
-- treats thermal and quantum phase fluctuations ✓
-- size limitations & moderate |U| ×

Maximum entropy method

for analytic continuation from "imaginary time" → real frequency

$$G(\mathbf{R};\tau) = -\int_{-\infty}^{\infty} d\omega \frac{e^{-\tau\omega}}{1+e^{-\beta\omega}} N(\mathbf{R},\omega).$$

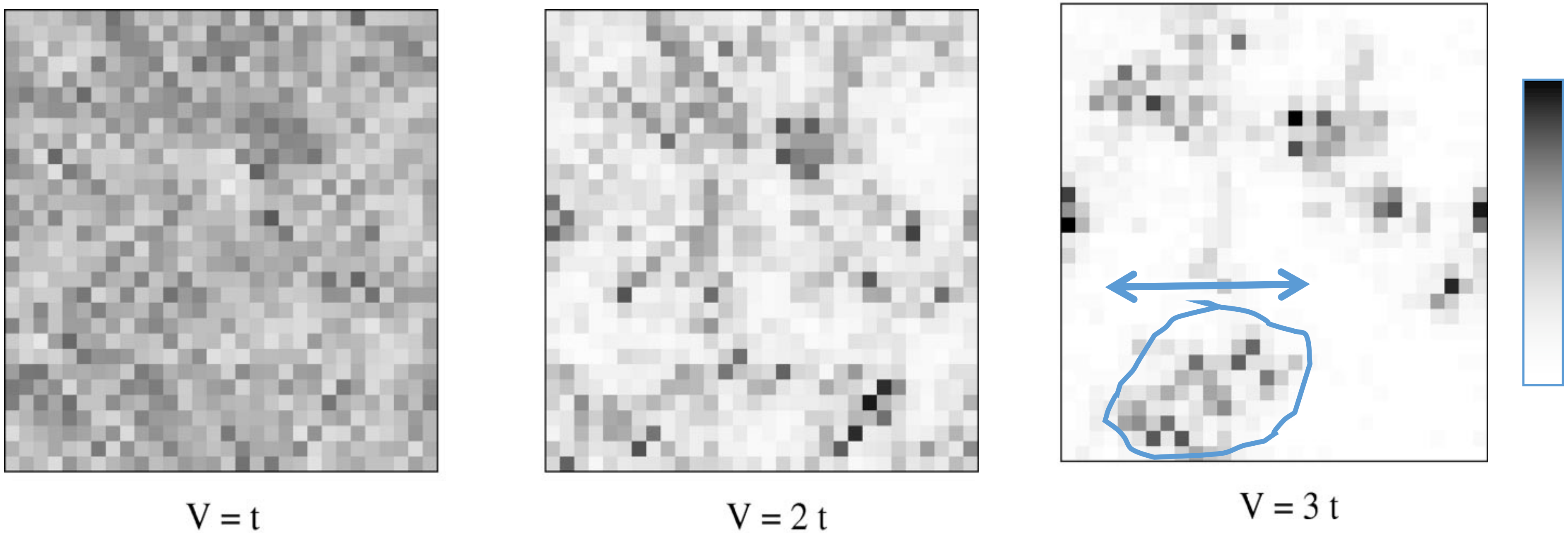
- Extensive checks on Max Ent: sum rules & moments, test functions;
- Gaps directly from "imaginary time" data
- Gap v/s gapless robust; sum rules robust; but not the large-w line-shape

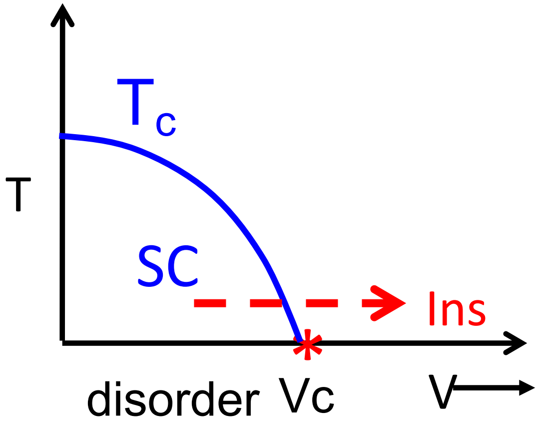
Real space maps of local pairing amplitude

$$\Delta(\vec{r}) = \langle c^{+}_{r\uparrow} c^{+}_{r\downarrow} \rangle$$

Emergent Granularity

Real space maps of $\Delta(\mathbf{R}) = \langle c_{\downarrow}(\mathbf{R}) \mathbf{c}_{\uparrow}(\mathbf{R}) \rangle$





increasing disorder

Patches $\sim \xi$

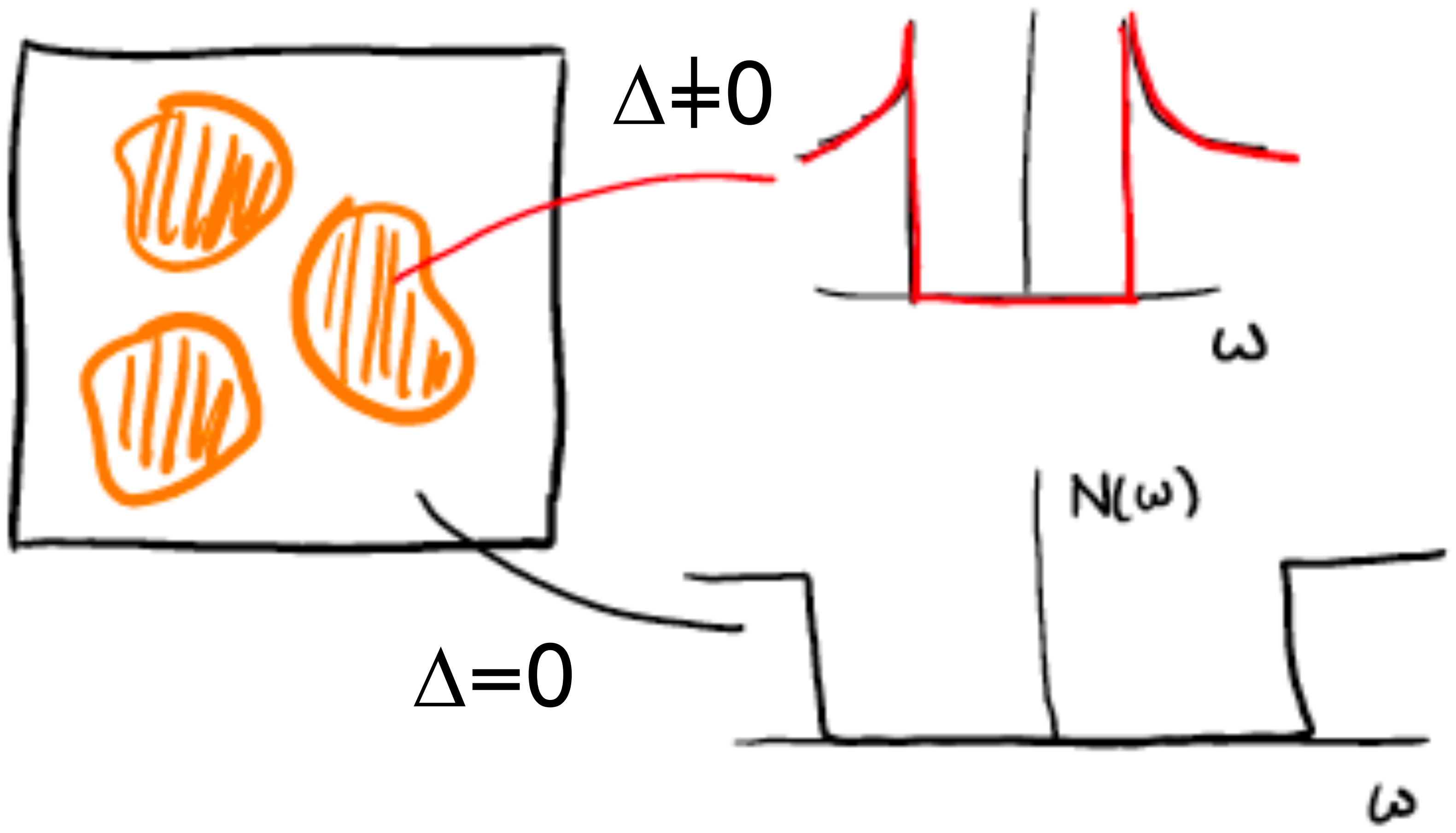
Structural disorder: atomic scale

Pairing of exact eigenstates; Bogoliubov de-Gennes inhomohenous mean field theory

Ghosal, Randeria & NT, PRL 81, 3940 (1998); PRB65, 014501 (2001)

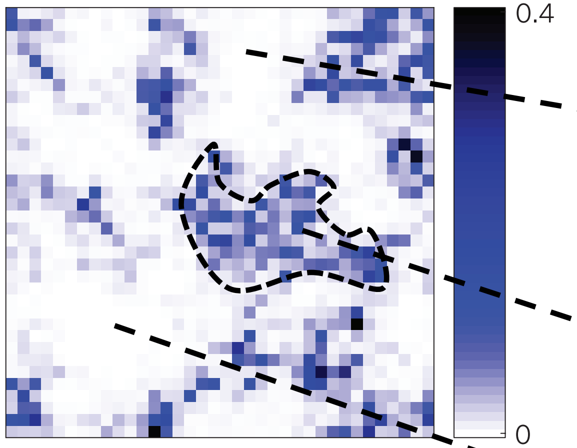
Can the local pairing amplitude be measured?

But…Local gap and local pairing amplitude are anti-correlated



Anticorrelation of gap and pairing amplitude

$\Delta(\mathbf{R})$

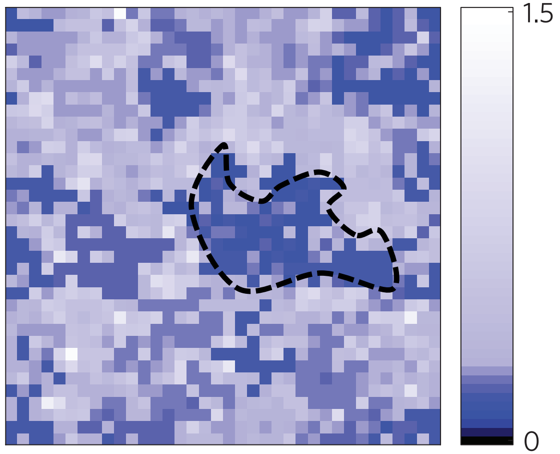


local pairing amplitude

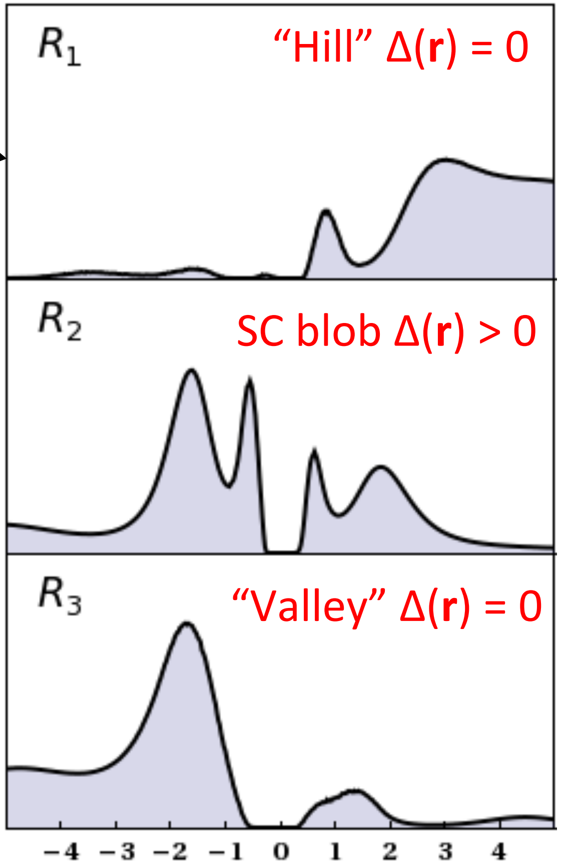
$\updownarrow$

energy gap in local DOS

$\omega_{\mathrm{dos}}(\mathbf{R})$



36x36, U=-1.5t, n=0.875, V=3t, T=0



large $\Delta(\mathbf{r})$ $\leftrightarrow$ small $\omega_{\mathrm{dos}}(\mathbf{r})$
small $\Delta(\mathbf{r})$ $\leftrightarrow$ large $\omega_{\mathrm{dos}}(\mathbf{r})$

Bouadim, Loh, Randeria & Trivedi Nature Phys. 7, 884 (2011)