

Coming back to Phil Nelson's comment of frequentists vs. bayesians

DID THE SUN JUST EXPLODE?
(IT'S NIGHT, SO WE'RE NOT SURE.)

THIS NEUTRINO DETECTOR MEASURES WHETHER THE SUN HAS GONE NOVA.

THEN, IT ROLLS TWO DICE. IF THEY BOTH COME UP SIX, IT LIES TO US. OTHERWISE, IT TELLS THE TRUTH.

LET'S TRY.

DETECTOR! HAS THE SUN GONE NOVA?

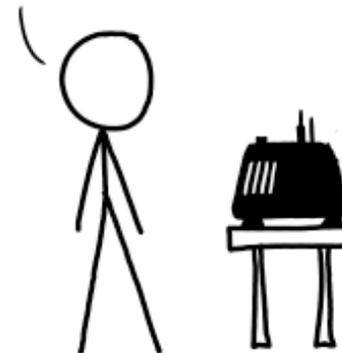
ROLL

YES.



FREQUENTIST STATISTICIAN:

THE PROBABILITY OF THIS RESULT HAPPENING BY CHANCE IS $\frac{1}{36} = 0.027$.
SINCE $p < 0.05$, I CONCLUDE THAT THE SUN HAS EXPLODED.



BAYESIAN STATISTICIAN:

BET YOU \$50 IT HASN'T.



Magnetic tweezers: how do they work, what are they good for, and how can their capabilities be expanded?

XX INTERNATIONAL SUMMER SCHOOL "NICOLAS CABRERA »

Biomolecules and Single Molecule Techniques

July 22, 2013



Delft University of Technology



Netherlands Organisation for Scientific Research

***Moving this
summer to
Munich:***

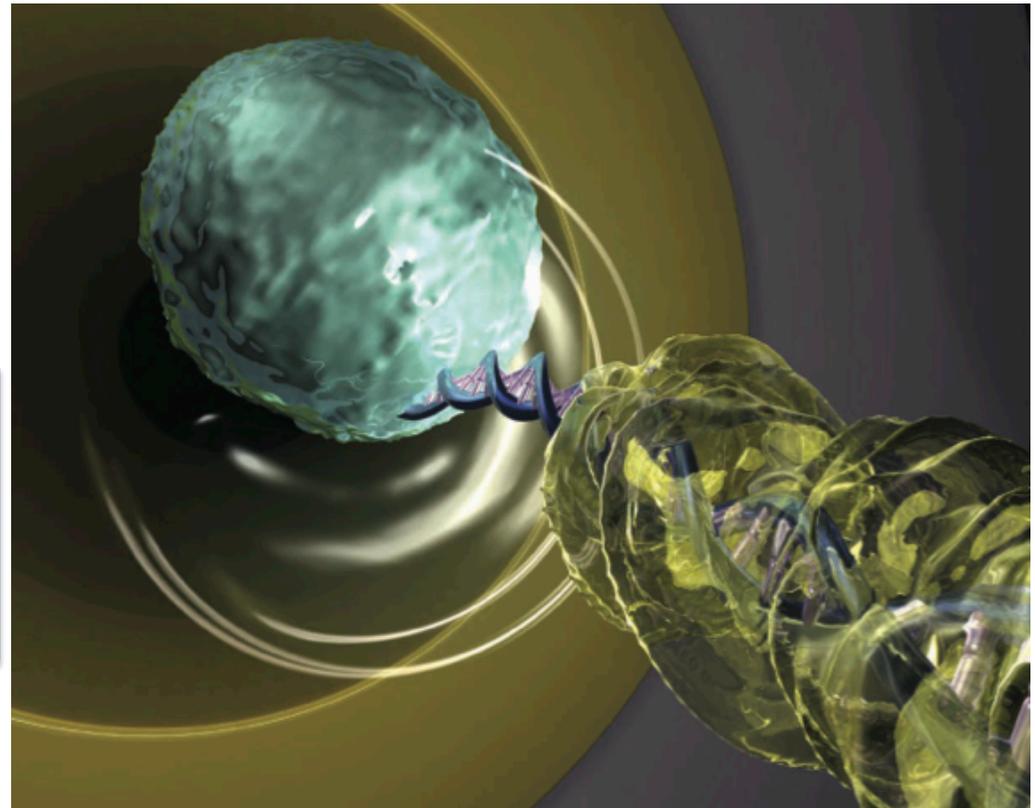


LUDWIG-
MAXIMILIANS-
UNIVERSITÄT
MÜNCHEN

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(Nynke Dekker Lab, Delft)

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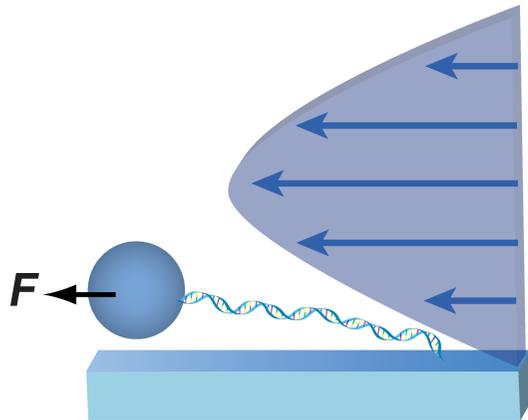
Lee, Lipfert, Sanchez, Wyman & Dekker *NAR* (2013)

Fernando's legacy in Delft lives on...

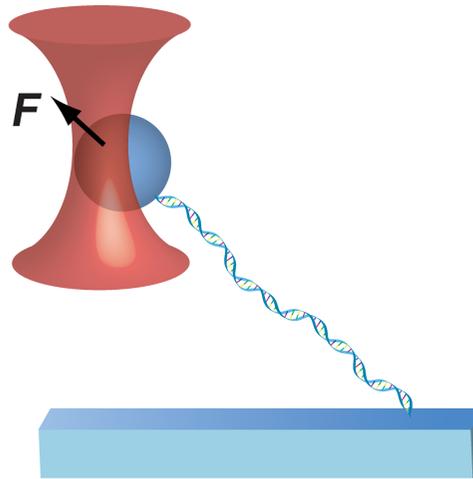


Single Molecule Manipulation Techniques

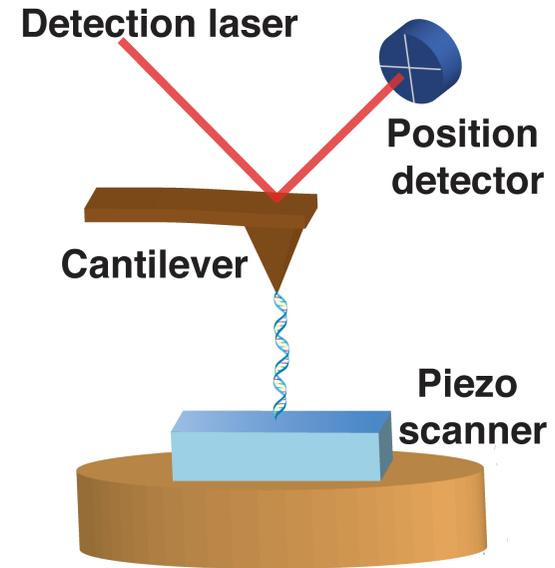
Flow stretch



Optical tweezers



AFM



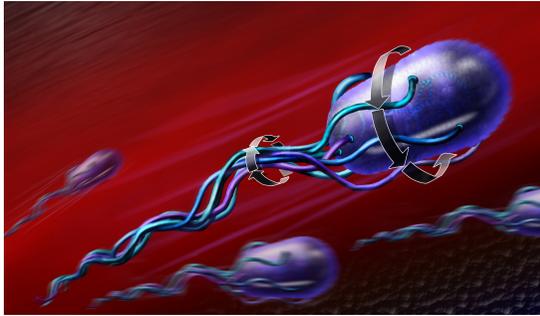
Adapted from: Dulin, Lipfert, Moolman & Dekker *Nature Reviews Genetics* (2013)

- In particular optical tweezers (OT) and atomic force microscopes (AFM) are commonly used to pull on single molecules
- Force manipulation has provided unique insights into mechanical properties and enzymatic activities

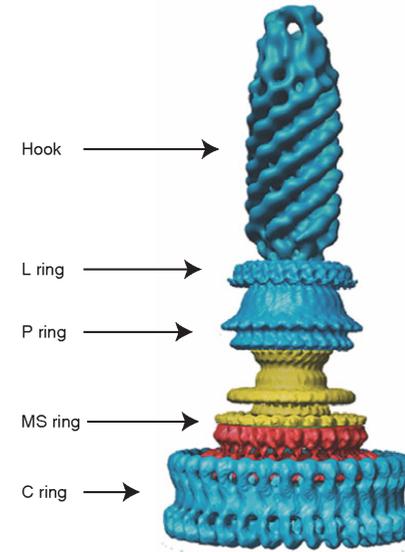
OT, AFM, and flow stretch measurements naturally operate in the space of **force and extension**.

Torque in Biology

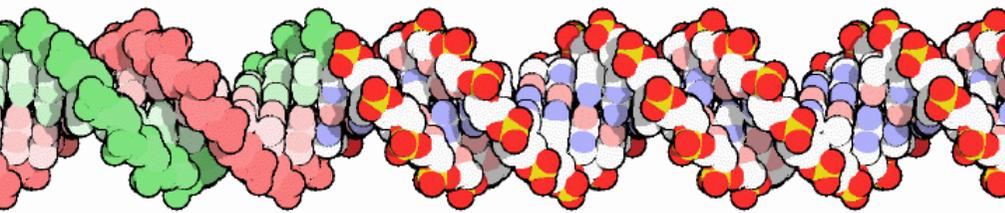
A classic example:
Bacterial flagella are rotary motors



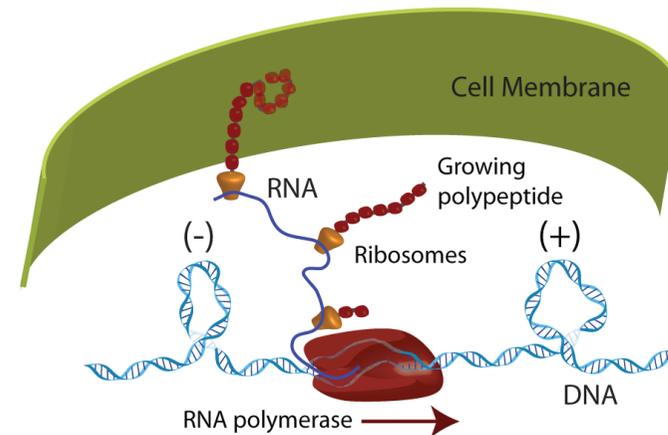
Sowa & Berry, *Quat, Rev. Biophys* (2008)



The helical nature of DNA has important consequences
for topology and torque generation in the cell

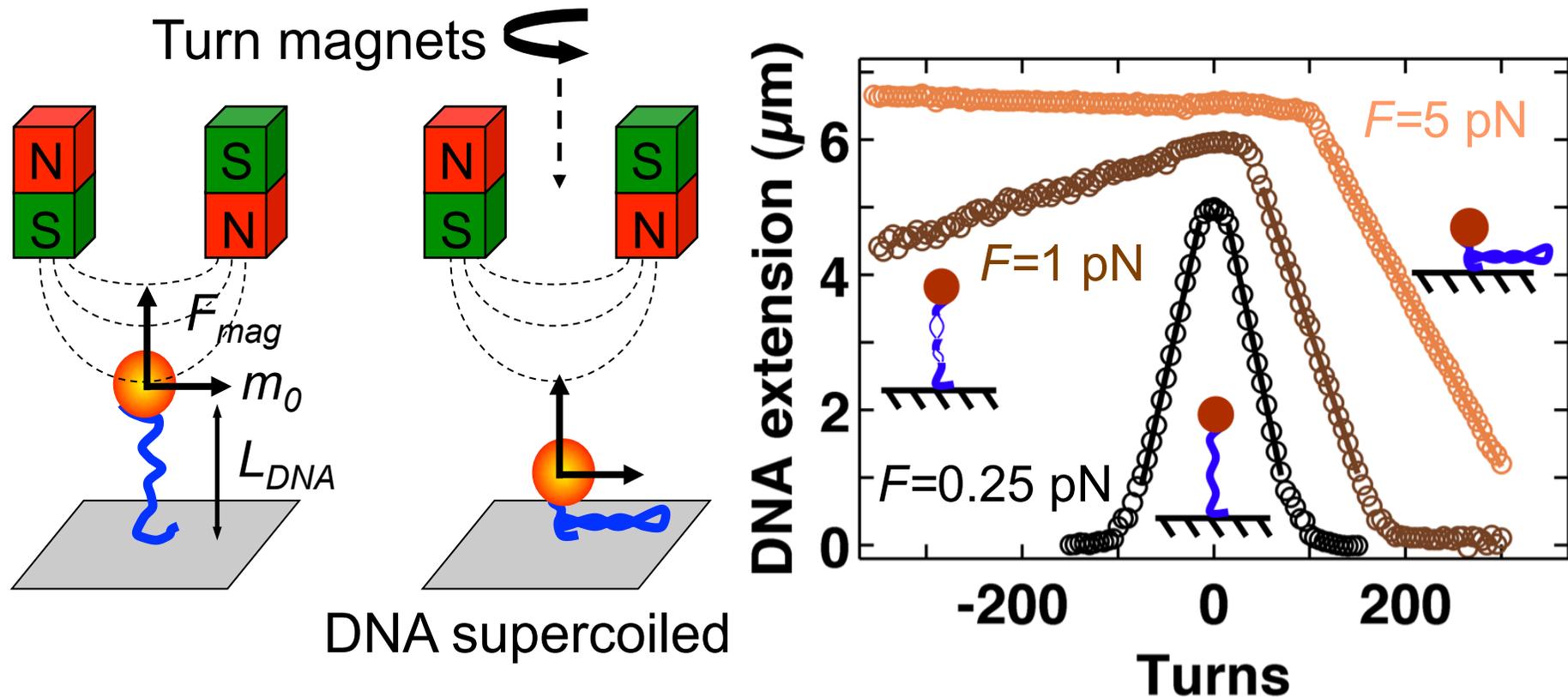


Liu & Wang, *PNAS* (1987)



Koster, Crut, *et al.*, *Cell* (2010)

(Super-)coiling DNA with Magnetic Tweezers



Magnetic tweezers allow us to apply both **forces** and **torques** to biological macromolecules.

Conventional MT: Strick, *et al.* *Science* (1996)

Figure adapted from: Lipfert, Koster, Vilfan, Hage & Dekker, *Methods Mol. Biol.* (2009)

Outline

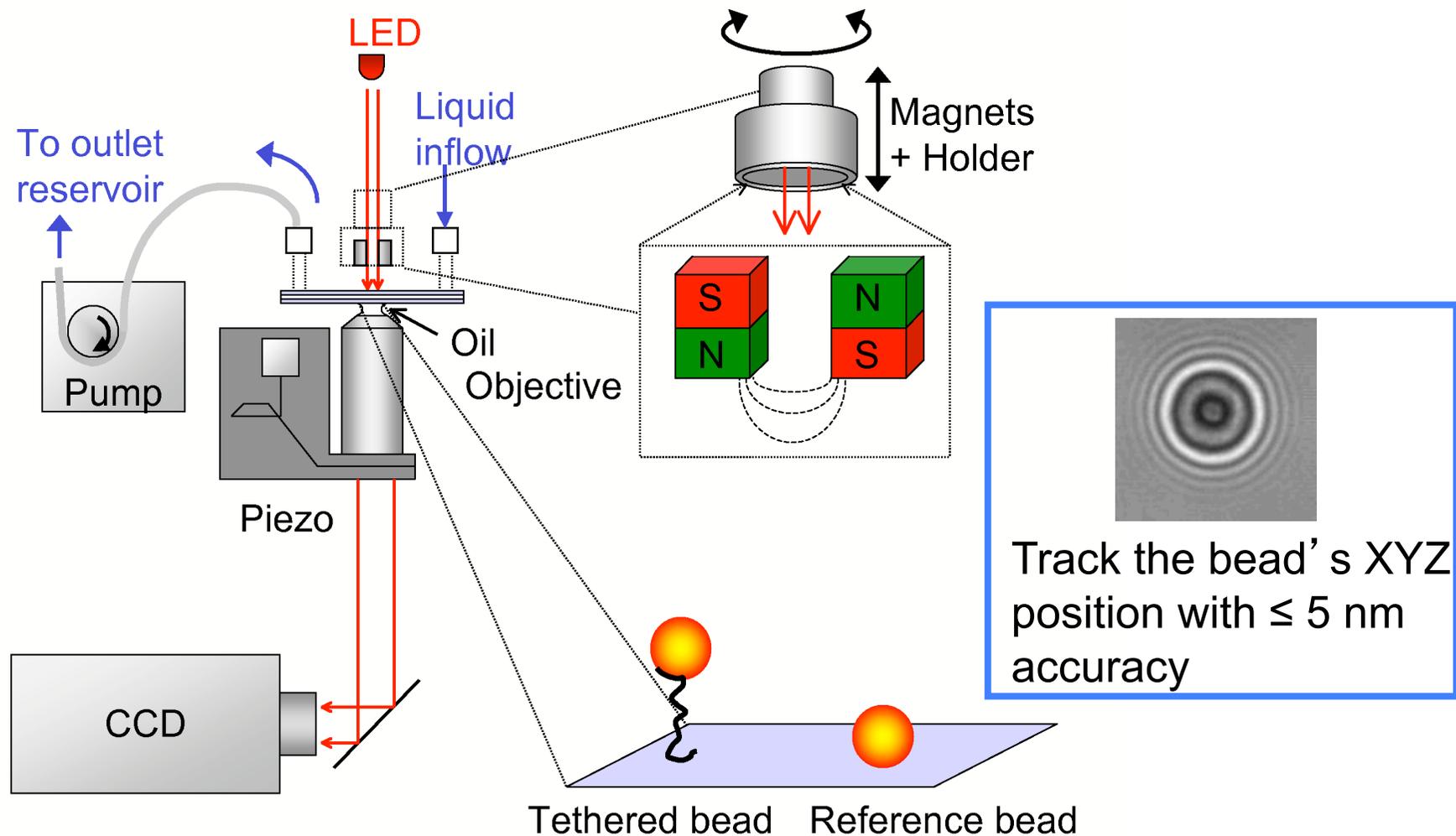
- How do (conventional) magnetic tweezers work?
 - *How do they apply forces?*
 - *How do they apply torques?*

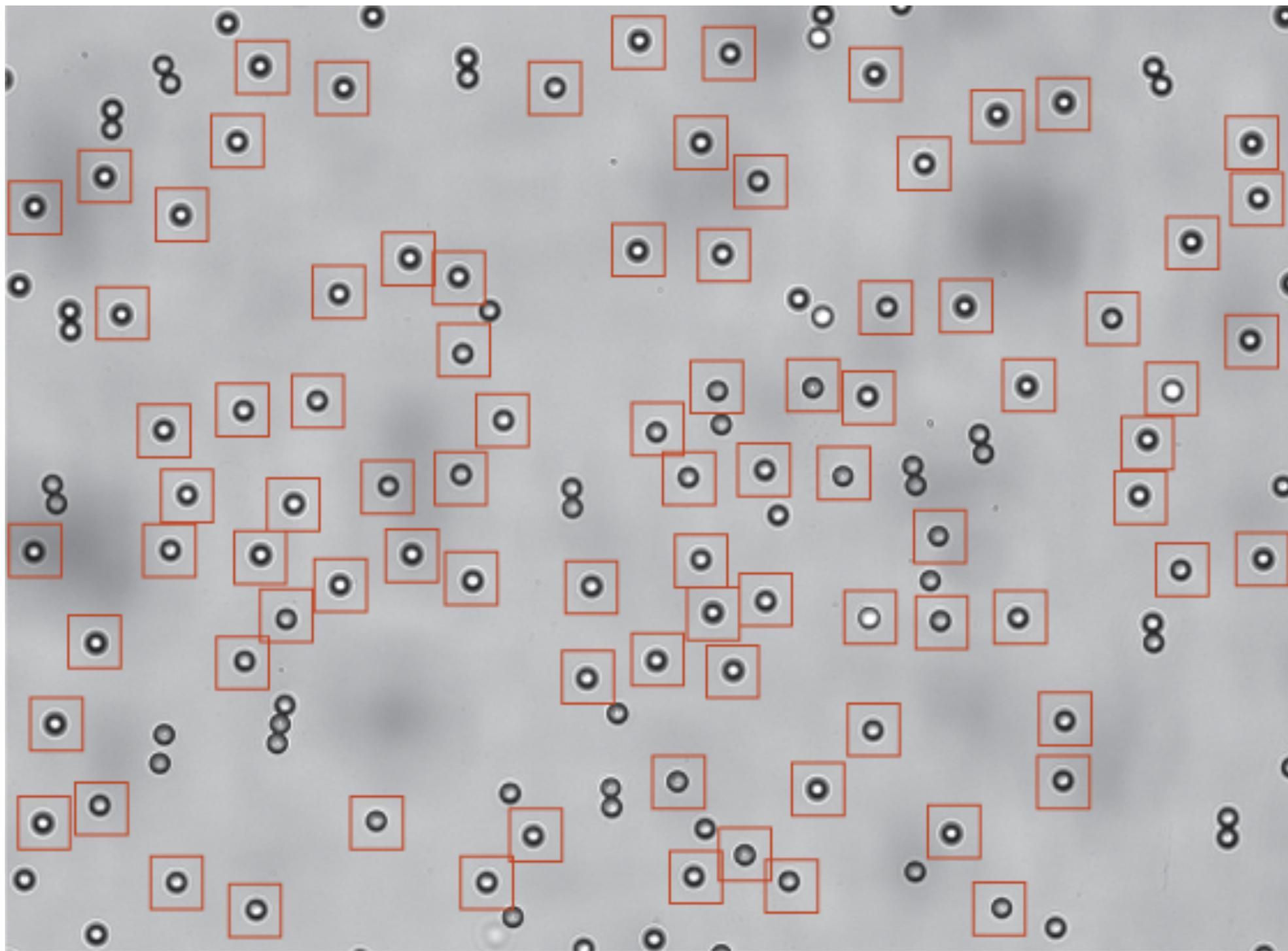
- Novel magnetic tweezers for direct measurements of single-molecule torque and twist
 - *Magnetic torque tweezers (MTT)*
 - *Freely-orbiting magnetic tweezers (FOMT)*

**The trouble with people is not their ignorance,
but that they know so much that ain't so.**

Mark Twain

Single Molecule Measurements with Magnetic Tweezers

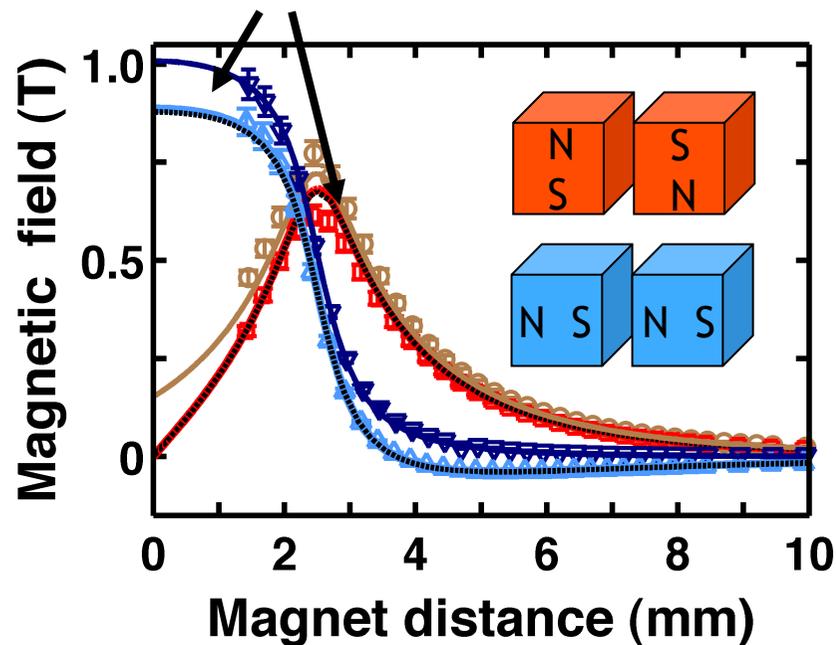




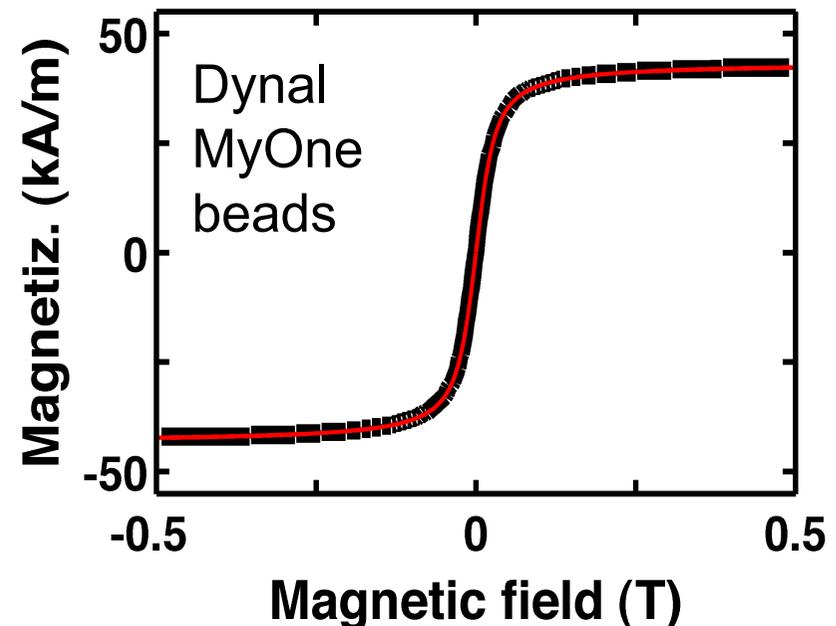
Modeling the force exerted by magnetic tweezers

The force on the beads is given by the negative gradient of the magnetic energy:
$$\vec{F}_{\text{mag}} = -\vec{\nabla} U_{\text{mag}} = \frac{1}{2} \vec{\nabla} \left(\vec{m}(\vec{B}) \cdot \vec{B} \right)$$

1) Calculate the magnetic field
With and without iron yoke

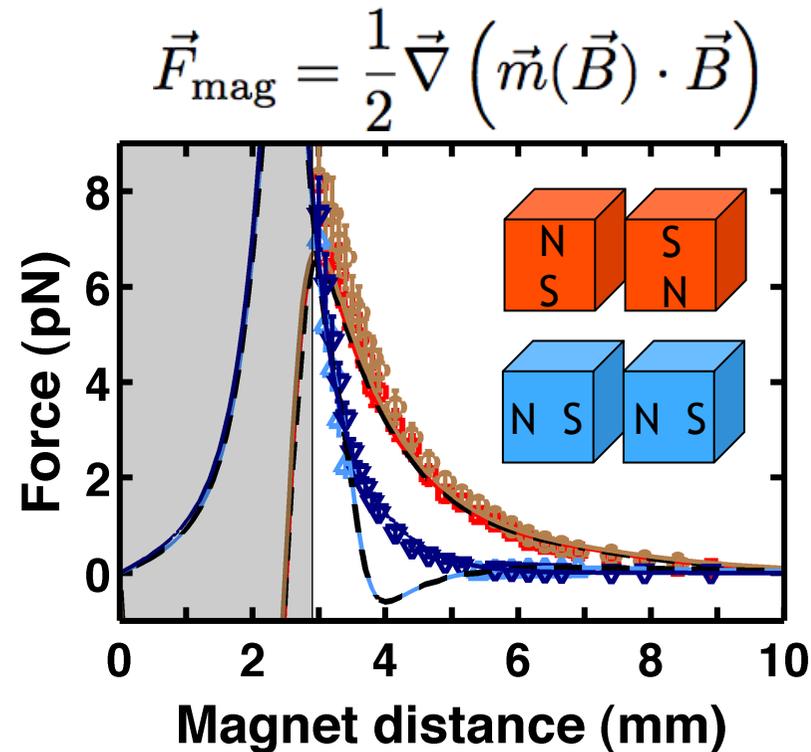


2) Consider the field dependent magnetization of the beads

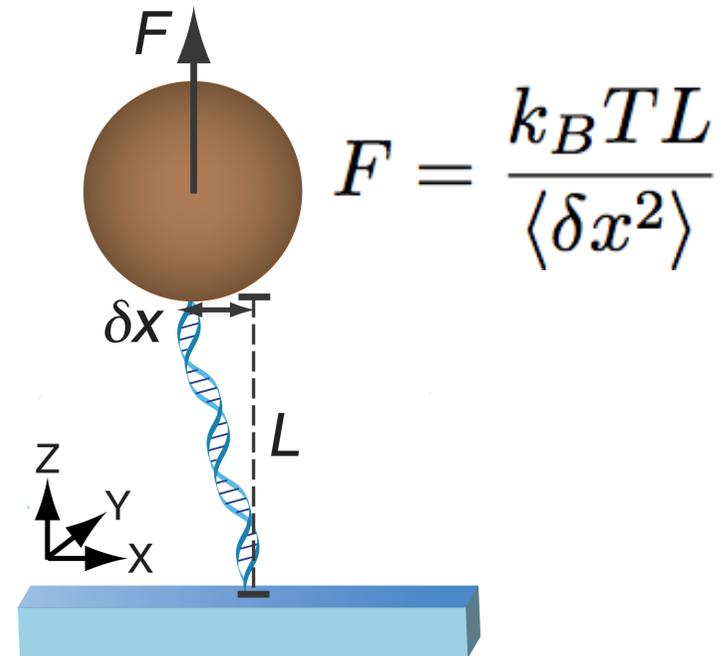


Measuring & modeling the forces in magnetic tweezers

3) Calculate the force exerted on the beads



4) Compare to forces determined from fluctuations*

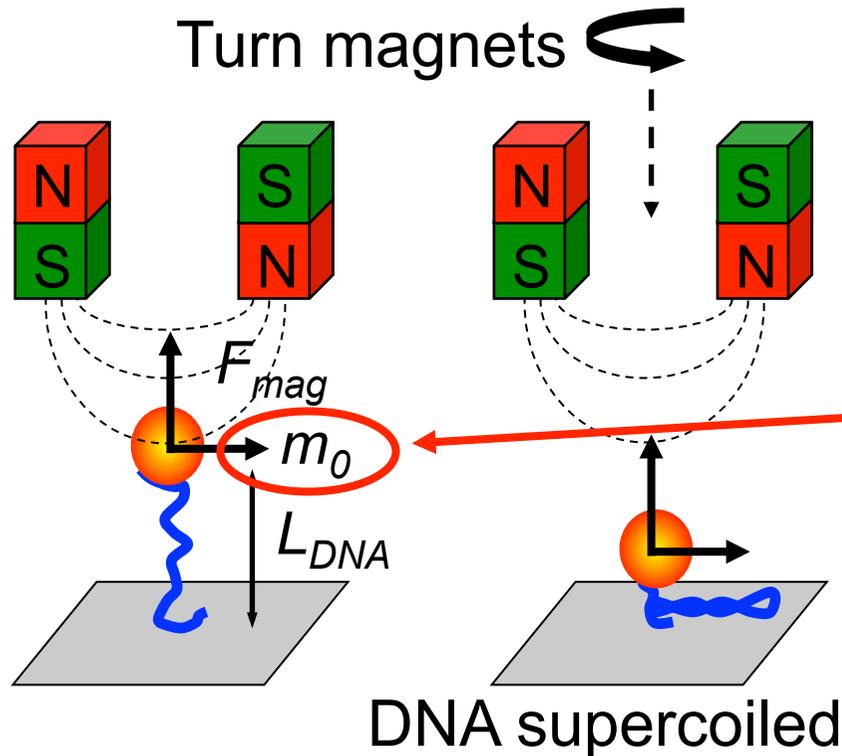


Predictive theory for MT from first principles

Lipfert, Hao & Dekker, *Biophys. J.*, (2009)

*Force calibration with spectral corrections:
Te Velthuis[†], Kerssemakers[†], Lipfert & Dekker
Biophys. J. (2010)

How do magnetic tweezers exert torque?



- For a perfectly paramagnetic bead, rotating the magnets would not apply any torque!

- There has to be a “special” or preferred direction in the bead to apply torques

Two models:

- Small constant & permanent magnetization component?

$$\vec{m}_0 = \text{const.}$$

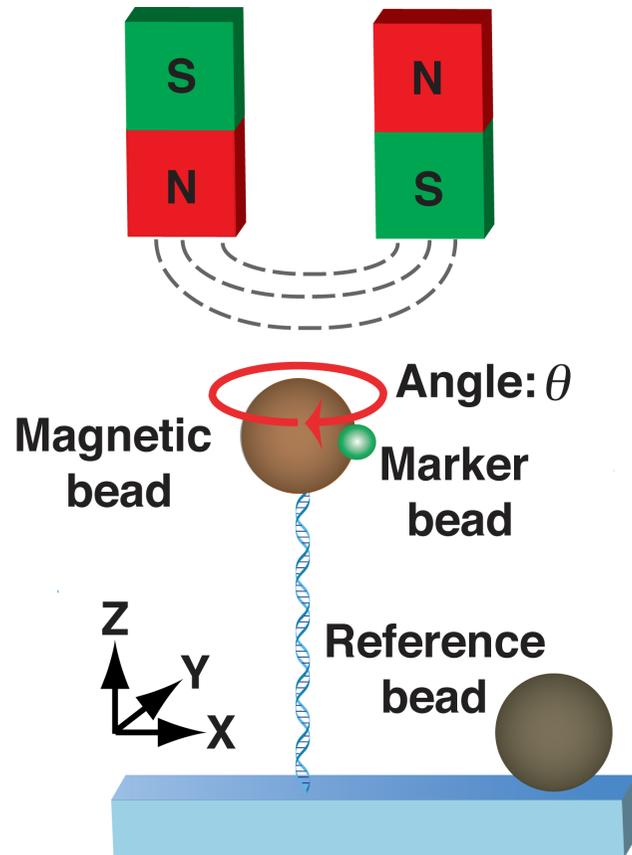
Janssen, Schellekens, Ommering, IJzendoorn & Prins
Biosens. Bioelectr. (2009)

- Soft or “easy” magnetization axis?

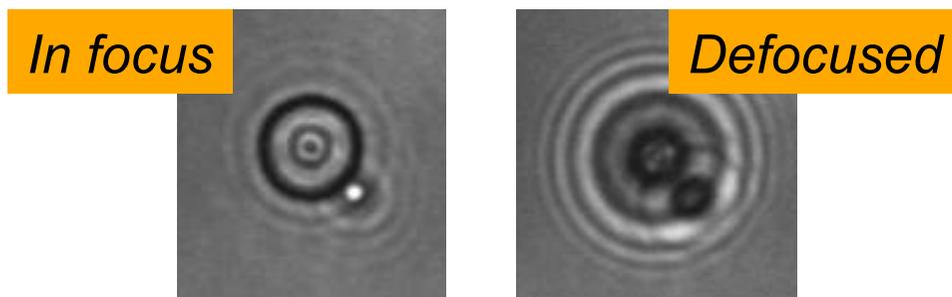
$$\vec{m}_0 = \vec{m}_0(\vec{B})$$

Theory: Normanno, Capitanio & Pavone *PRA* (2004)
 Indirect experimental evidence: Klaue & Seidel *PRL* (2009)

Directly monitor bead rotation in MT

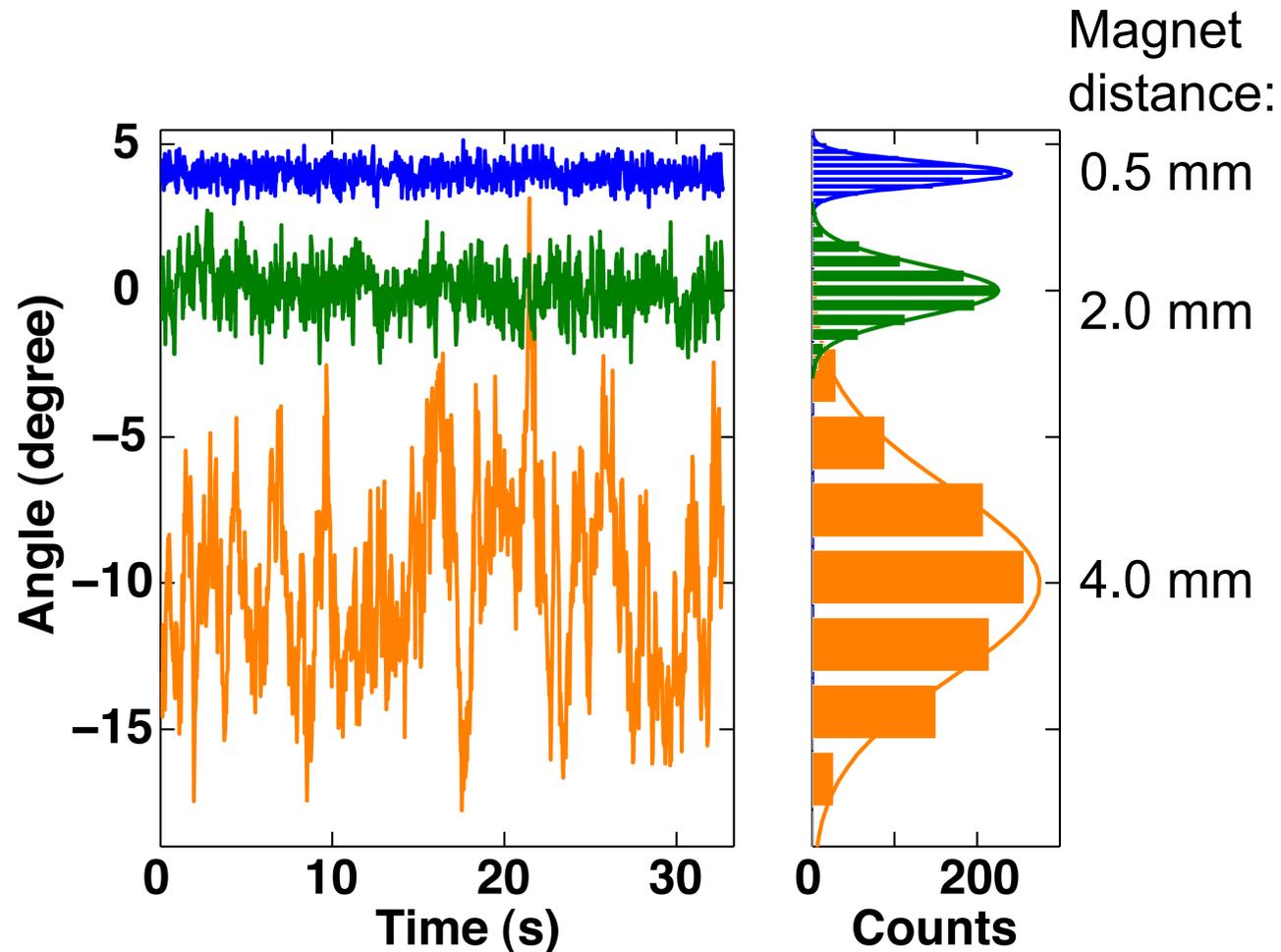


- Attach a (non-magnetic) fiducial marker bead to the magnetic bead



- Implement angular tracking protocol to monitor bead's rotation as well as (x,y,z) -position

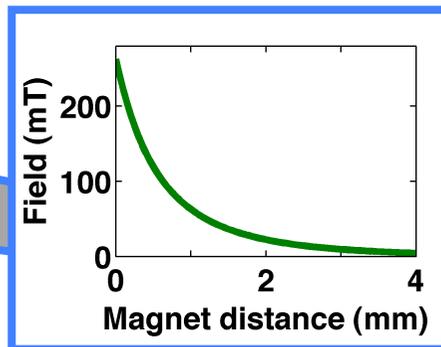
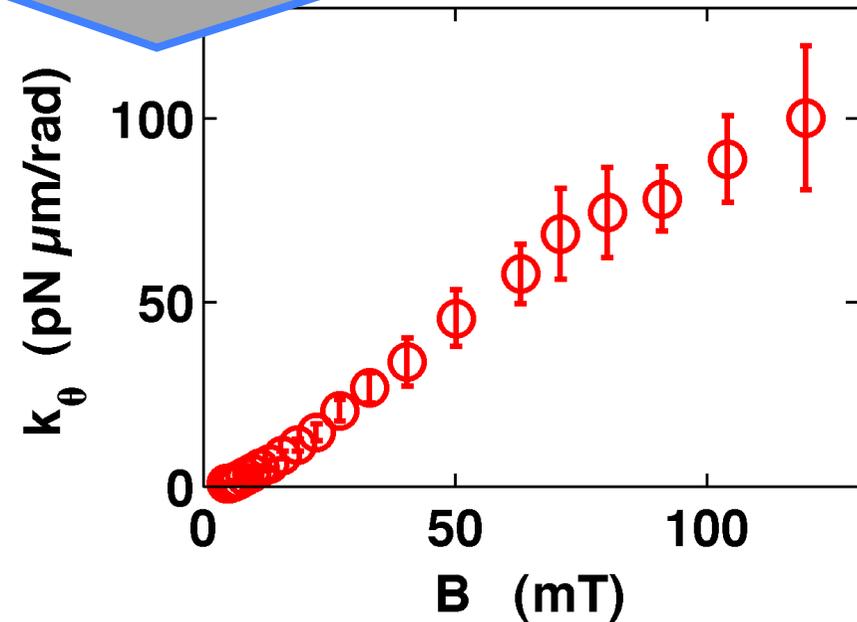
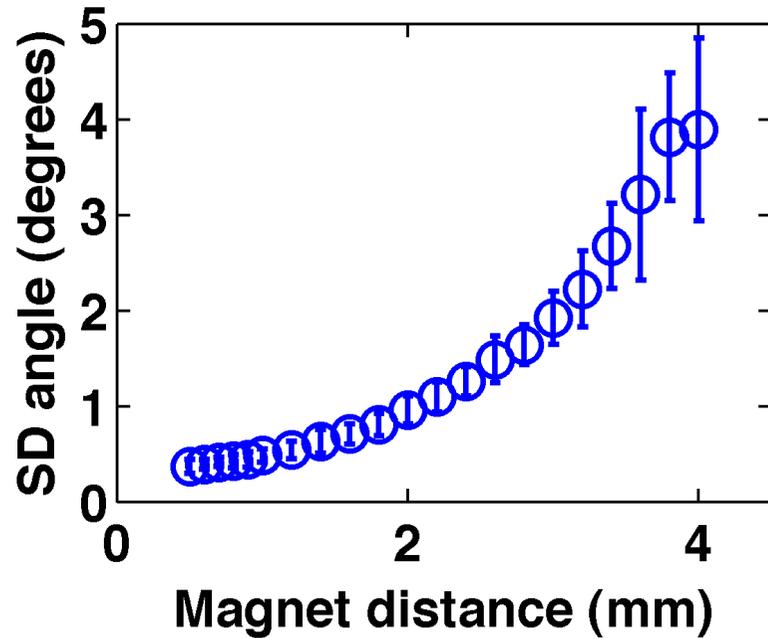
Measure the rotational fluctuations as a function of magnet position



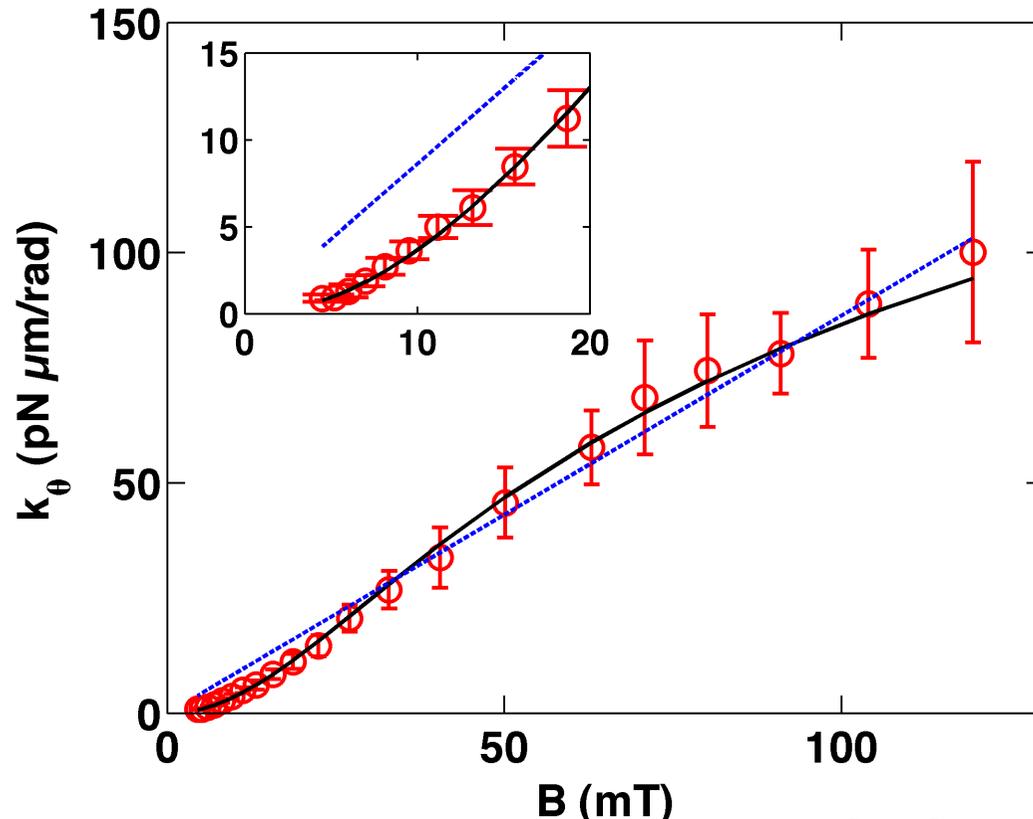
Traces offset for clarity. M270 beads tethered to 8 kbp DNA in PBS buffer. Horizontal magnets, 1 mm gap, with iron yoke.

Rotational trap stiffness as a function of magnetic field

$$k_{\theta} = \frac{k_B T}{\sigma_{\theta}^2}$$



The experimental data support a “soft axis” model



Model with permanent magnetization component:

$$k_{\theta} = m_0 \cdot B$$

- Fit m_0 (= dipole moment)
- reduced $\chi^2 = 59$, $p \ll 0.01$

Soft magnetization axis model:

- reduced $\chi^2 = 0.13$, $p = 0.99$

$$k_{\theta} = N \cdot V \cdot \left(BM \left(1 + \frac{B^2 M^2}{(C + BM)^2} - \frac{2BM}{(C + BM)} \right) + \frac{B^2 C M^2}{(C + BM)^2} \right)$$

Theory: Normanno, Capitanio & Pavone *PRA* (2004)

- Fit: $C = 2990 \text{ J/m}^3$ (crystalline anisotropy constant)
- and $(N \cdot V) = 0.06 \text{ } \mu\text{m}^3$ (~effective size of magnetic grains)

Lipfert, *et al.*, *in preparation*

**The trouble with people is not their ignorance,
but that they know so much that ain't so.**

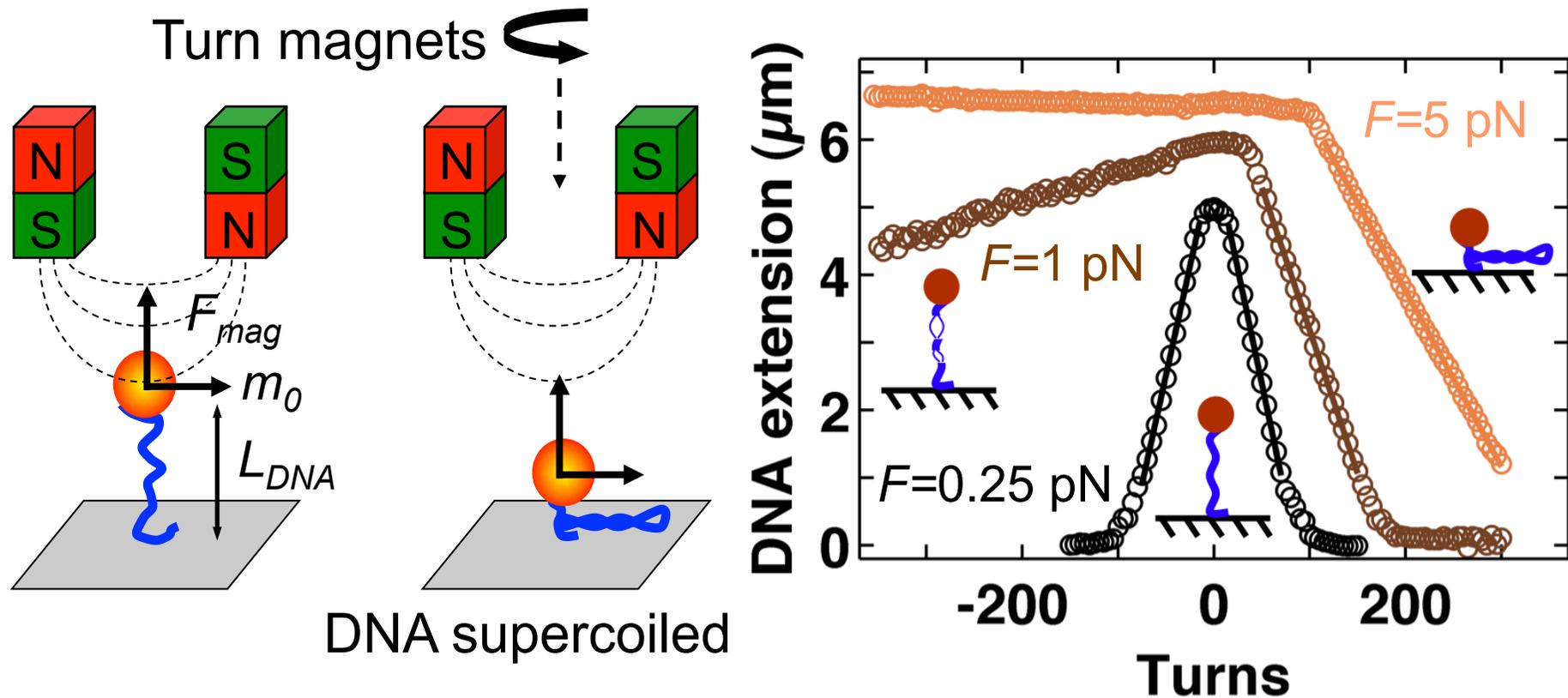
~~Mark Twain~~

Josh Billings

Outline

- How do (conventional) magnetic tweezers work?
 - *How do they apply forces?*
 - *How do they apply torques?*
- Novel magnetic tweezers for direct measurements of single-molecule torque and twist
 - *Magnetic torque tweezers (MTT)*
 - *Freely-orbiting magnetic tweezers (FOMT)*

Conventional Magnetic Tweezers



However, current MT

- **Do not track** the bead's rotation directly
- **Do not permit** to measure the applied torque

Conventional MT: Strick, *et al.* *Science* (1996)

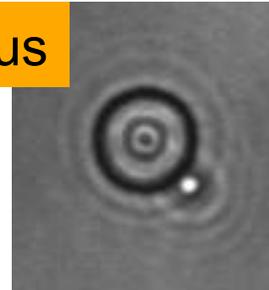
Figure adapted from: Lipfert, Koster, Vilfan, Hage & Dekker, *Methods Mol. Biol.* (2009)

Magnetic Torque Tweezers

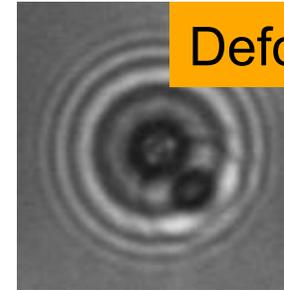
Two changes compared to “conventional” MT:

- 1) Attach a fiducial marker bead and implement angular tracking

In focus

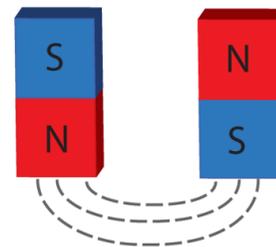


Defocused

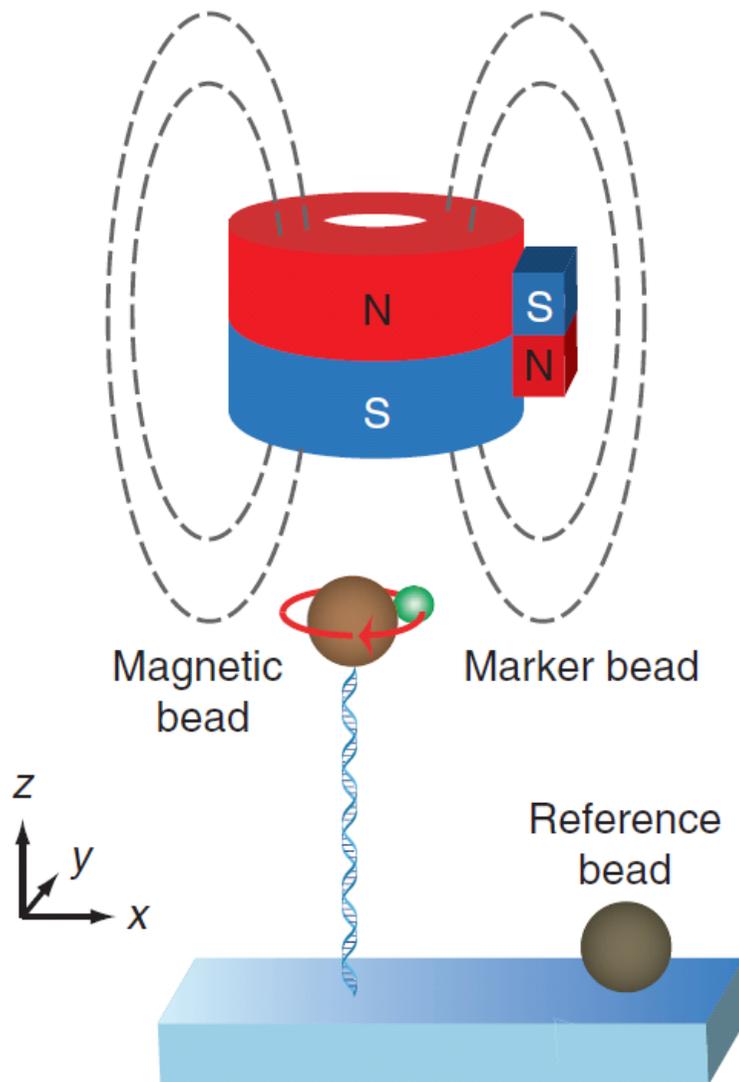
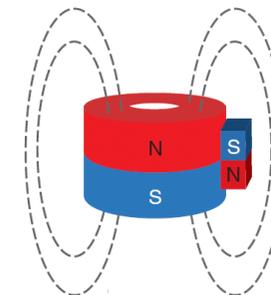


Simultaneously track X, Y, Z, and angle (& force and torque)

- 2) Modify the magnet geometry to lower the torsional trap stiffness



vs.

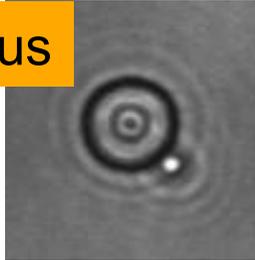


Lipfert, Kerssemakers, Jager & Dekker, Nature Methods (2010)
Lipfert[†], Kerssemakers[†], Rojer & Dekker, Rev. Sci. Instrum. (2011)

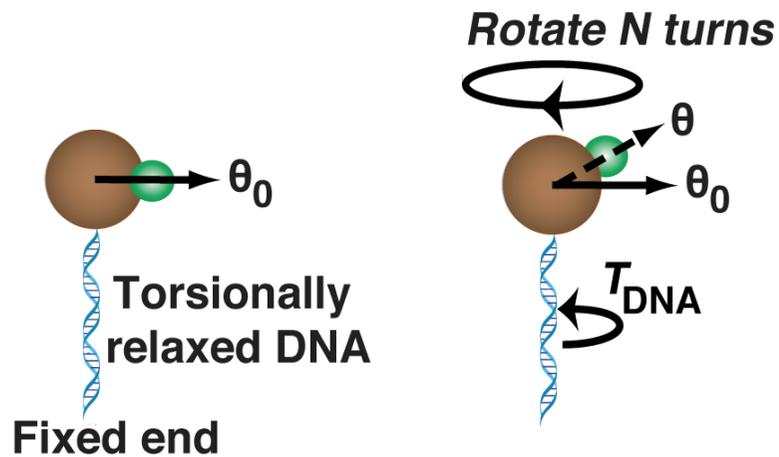
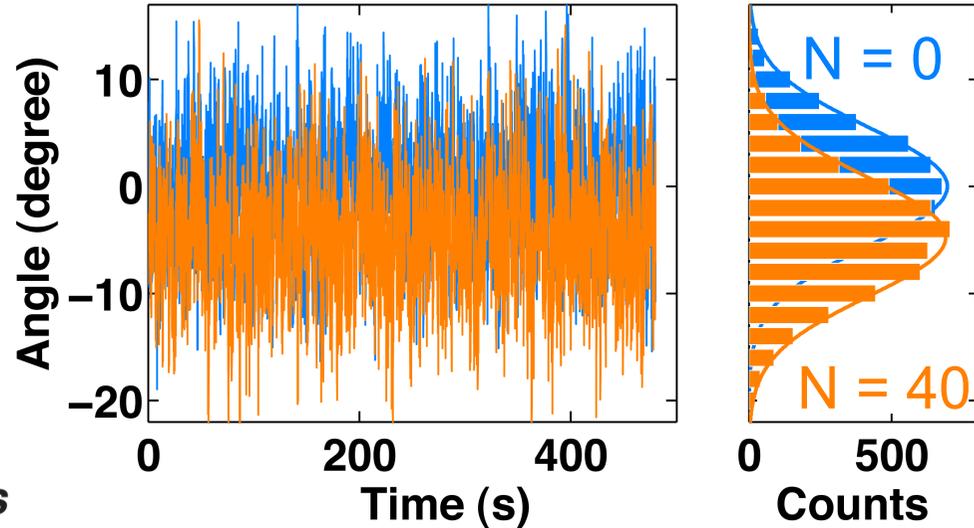
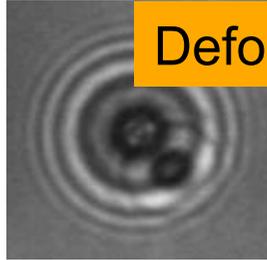
Principle of Magnetic Torque Tweezers

Fiducial marker beads facilitate angle tracking in addition to (x,y,z)

In focus



Defocused



Restoring torque of the tether gives a shift in the equilibrium position:

$$\tau_{\text{DNA}} = k_{\theta} \langle \theta - \theta_0 \rangle$$

$$\langle \theta \rangle = \theta_0$$

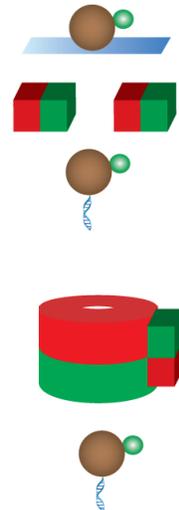
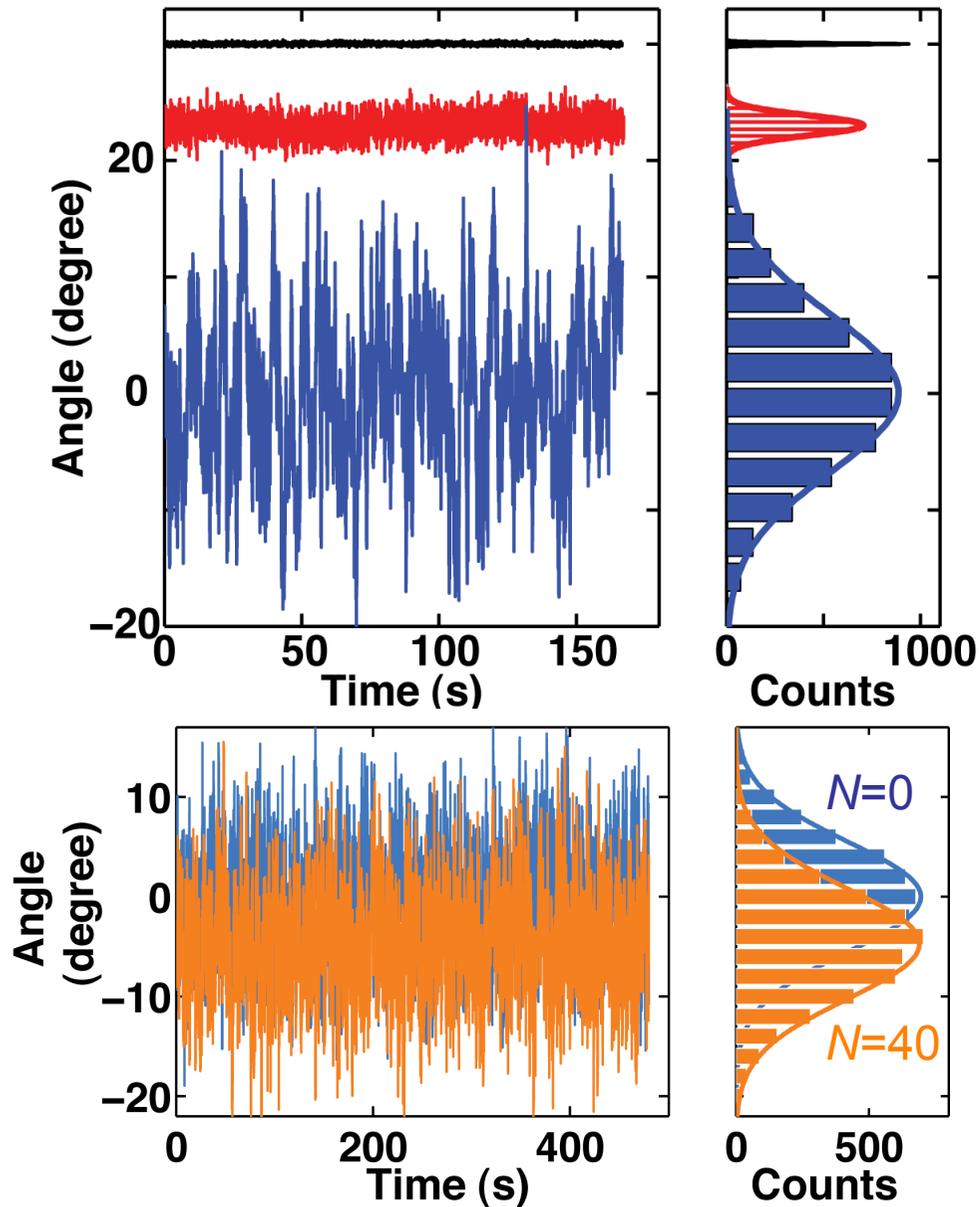
Mean (equilibrium) angle

$$k_{\theta} = \frac{k_B T}{\langle (\theta - \theta_0)^2 \rangle}$$

Calibrate the trap stiffness from the fluctuations, using equipartition

Angular Resolution and Magnet Geometry

Angular traces from image analysis



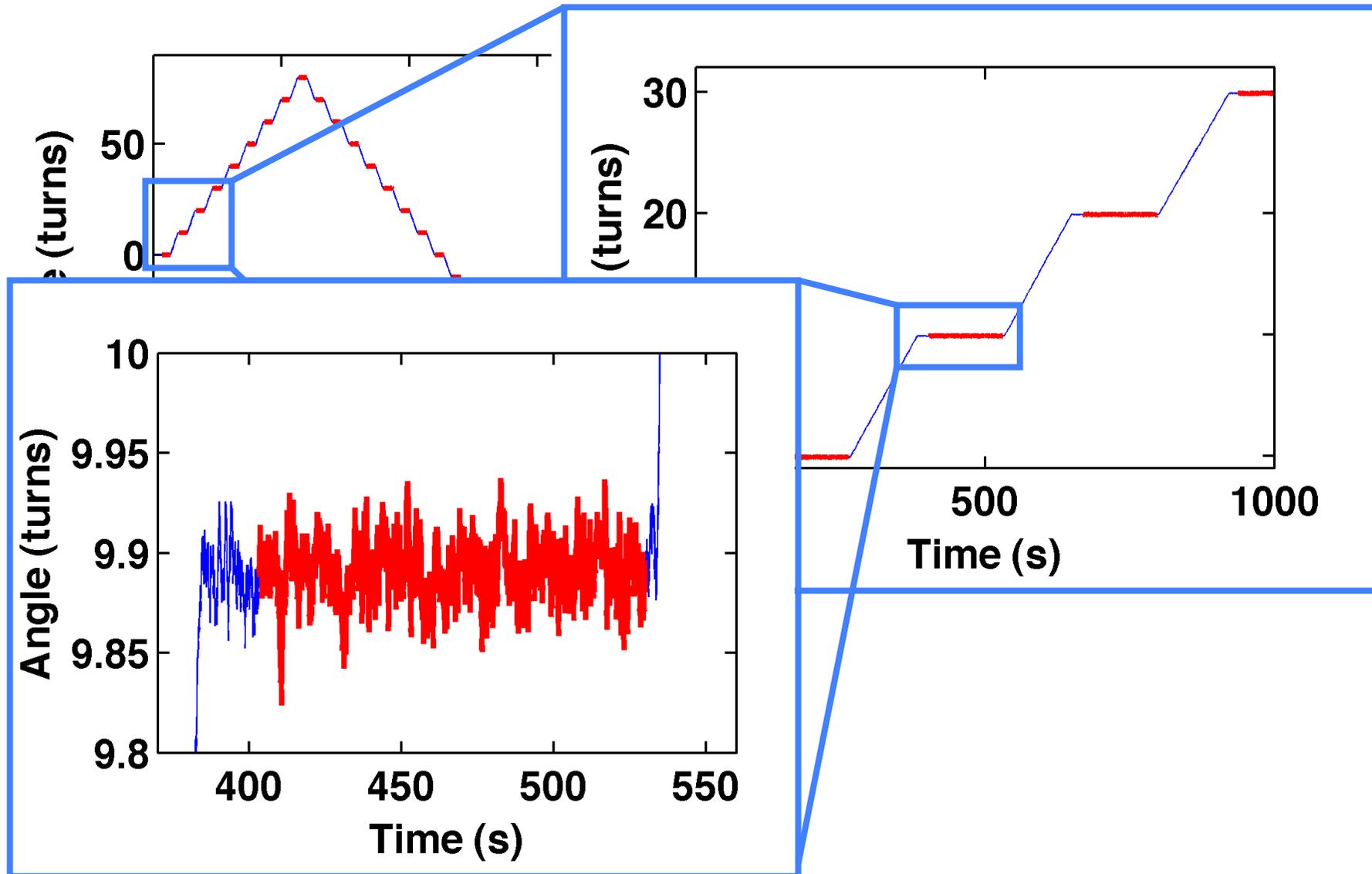
Surface attached bead:
 $\sigma_{\theta} \approx 0.1^{\circ}$

Conventional magnetic tweezers:
 $\sigma_{\theta} \approx 1^{\circ}$
 \Rightarrow Trap stiffness k_{θ} is 10^4 - 10^5 pN·nm/rad
 \Rightarrow Expected DNA signal $\langle \theta - \theta_0 \rangle \leq 0.1^{\circ}$

Torque tweezers:
 $\sigma_{\theta} \approx 5$ - 10°
 \Rightarrow Trap stiffness k_{θ} is 100-500 pN·nm/rad

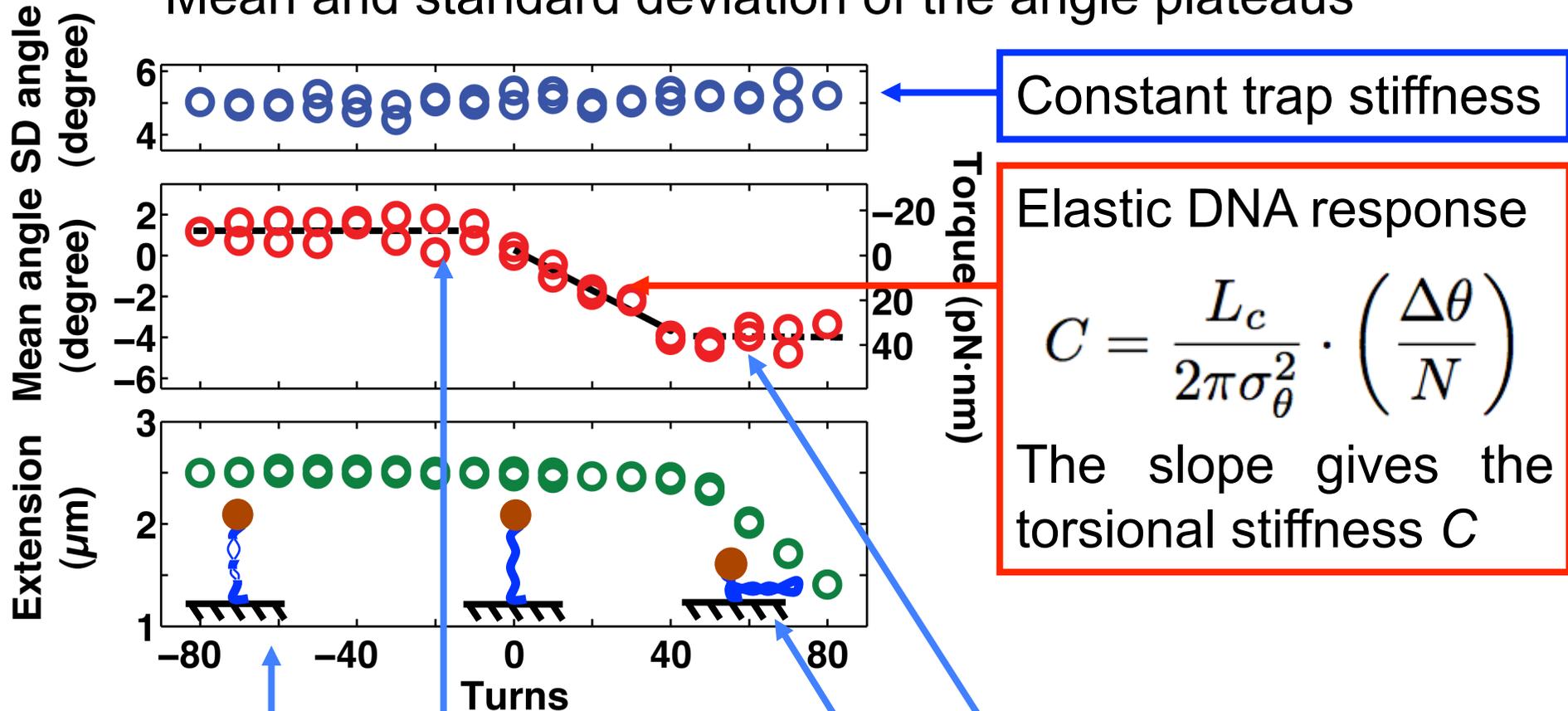
“Twist curve” measurement

Our strategy for measuring the torsional stiffness of DNA



Torque Measurement for DNA

Mean and standard deviation of the angle plateaus

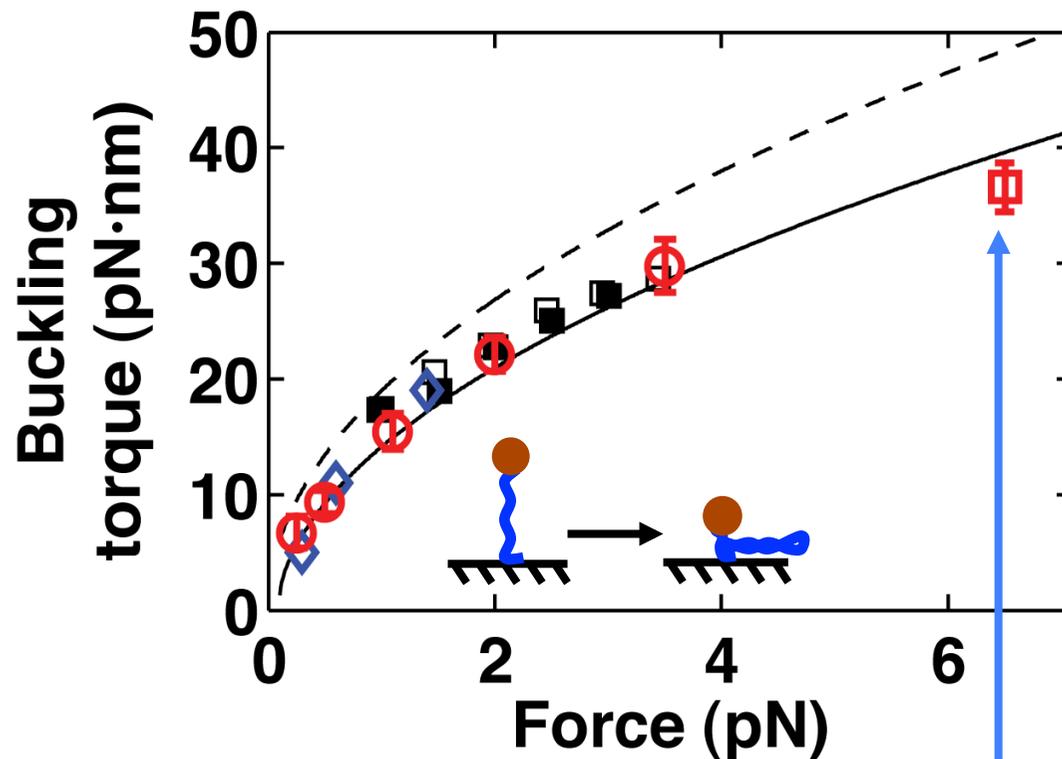


@ negative turns, the torque saturates as DNA denatures
 $\tau_{\text{denat}} = -10 \pm 1 \text{ pN}\cdot\text{nm}$

@ positive turns, the torque saturates as the DNA buckles and plectonemes form

Measurement for 8 kbp DNA in PBS buffer; $F = 3.5 \text{ pN}$

DNA Buckling Torque



Magnetic torque tweezers
(this work)

Optical torque tweezers
(Wang lab)

Open squares: 2.2 kbp DNA
Filled squares: 4.2 kbp DNA
Forth, et al. *PRL* (2008)

Nanorod probe
(Sun lab)

Celedon, et al. *Nano Lett.* (2009)

- - - Simple elastic model
Strick, et al. *Rep. Prog. Phys.* (2003)

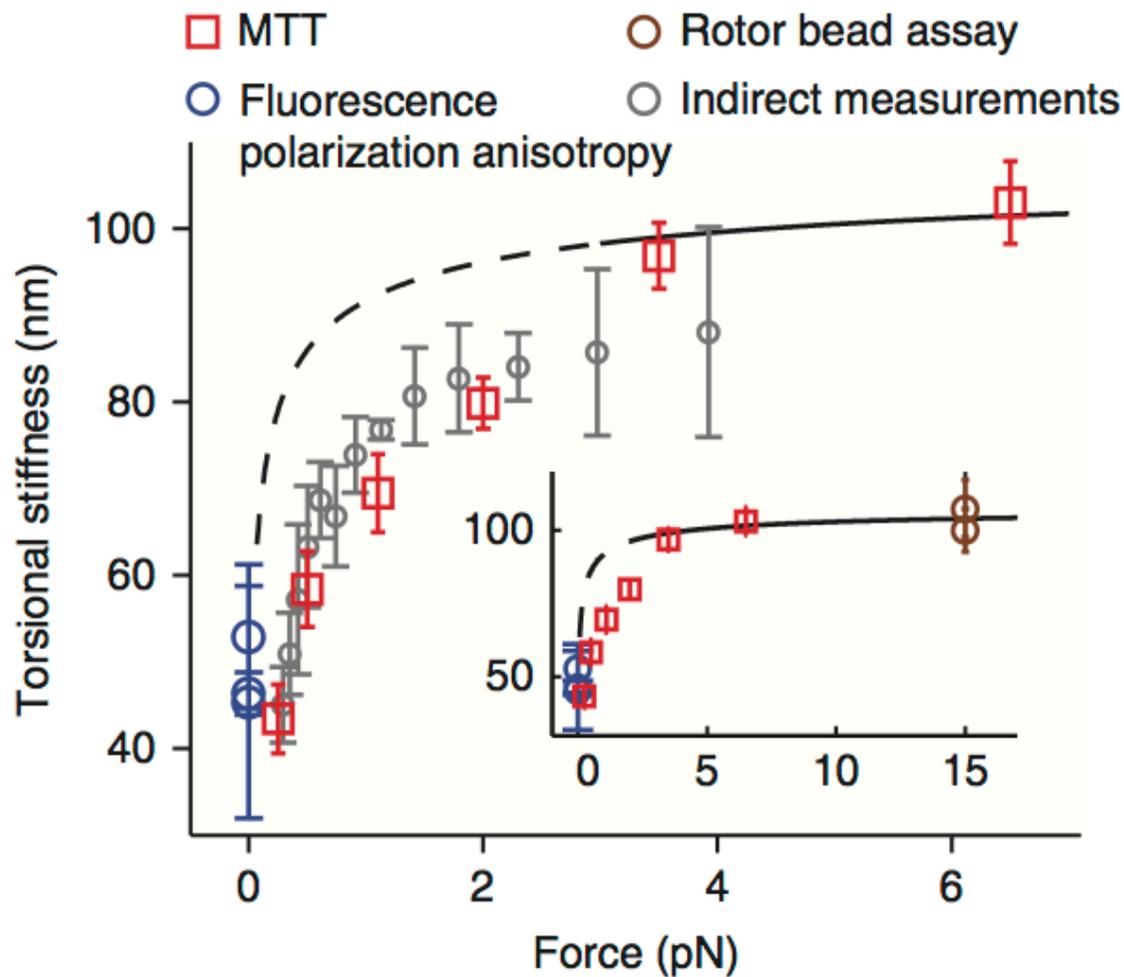
— Marko model
Marko *PRE* (2007)

@ forces > 6 pN, (B-form) DNA does not buckle, but undergoes a transition to P-DNA. We measure: $\tau_{B \rightarrow P} = 36 \pm 2 \text{ pN}\cdot\text{nm}$

See also: Sheinin et al. *PRL* (2008) and Bryant et al. *Nature* (2003)

MTT measurements are for 8kbp DNA in PBS buffer. Red symbols are the mean and SEM of 5-10 independent molecules; Ref: Lipfert, Kerssemakers, Jager & Dekker, *Nature Methods* (2010)

DNA Torsional Persistence Length



Magnetic torque tweezers

Rotary bead tracking @ high F
(Bustamante lab)

Bryant *et al. Nature* (2003)

FPA measurements on linear DNA \Rightarrow zero force

Fujimoto & Schurr *Nature* (1990)

Selvin *et al. Science* (1992)

Heath *et al. JMB* (1996)

Indirect measurement by thermodynamic integration

Mosconi *et al. PRL* (2009)

Model by Moroz & Nelson

Moroz & Nelson *PNAS* (1997)

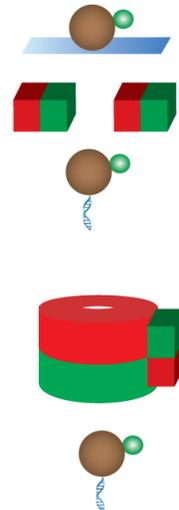
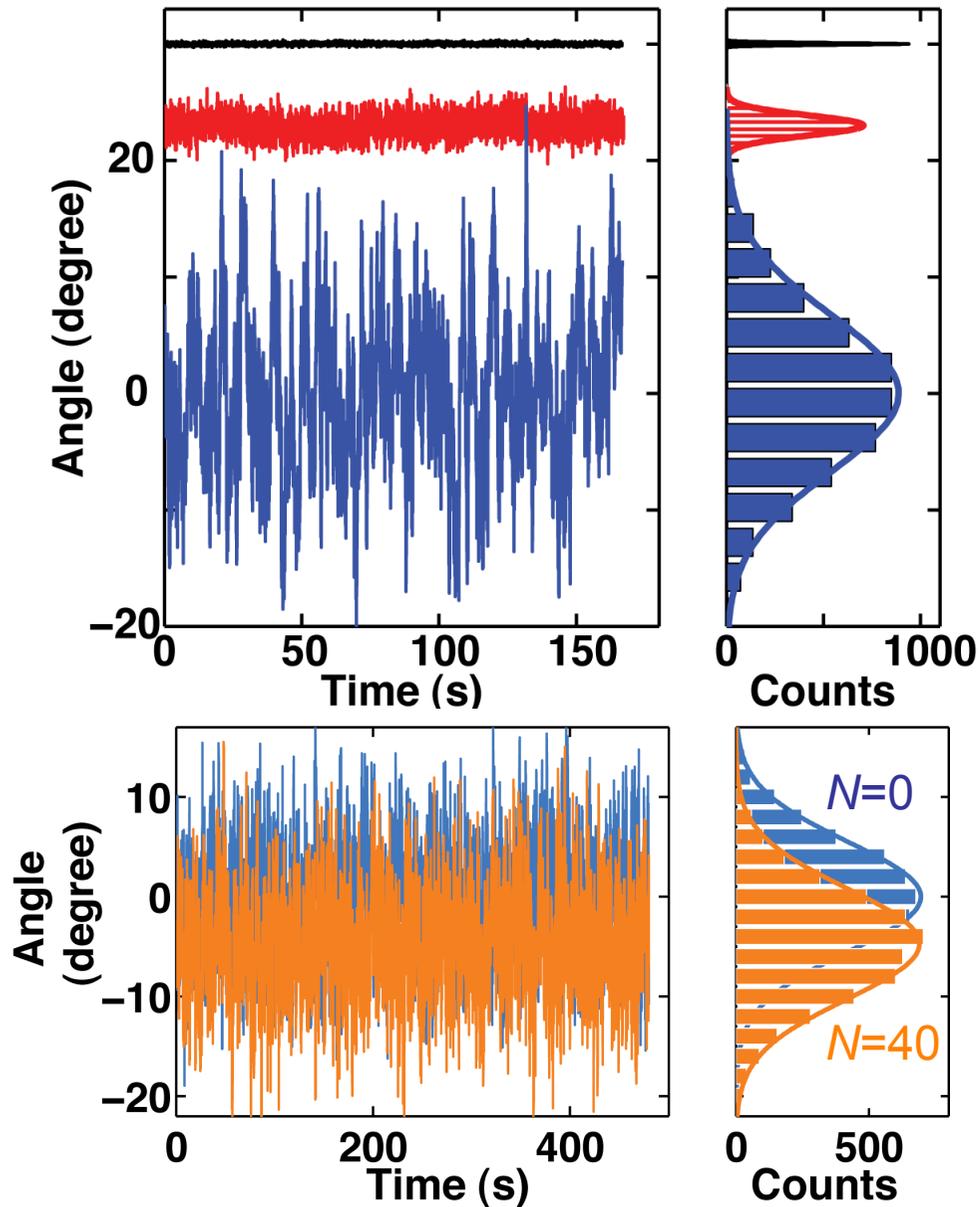
MTT measurements cover the entire force range from “high” to effectively zero

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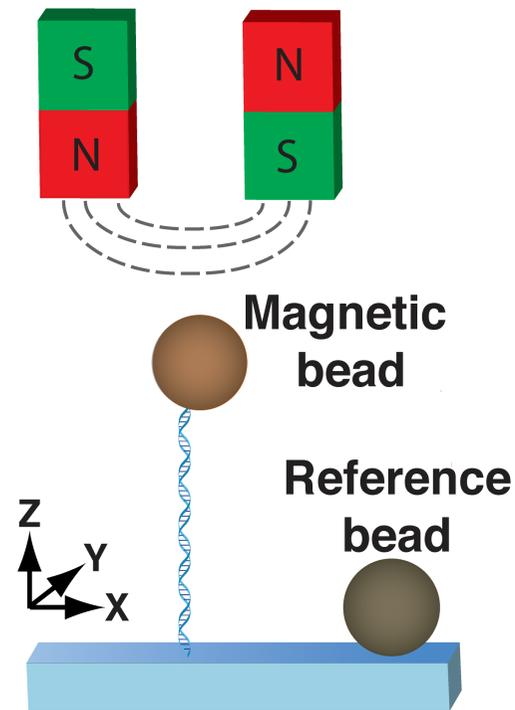
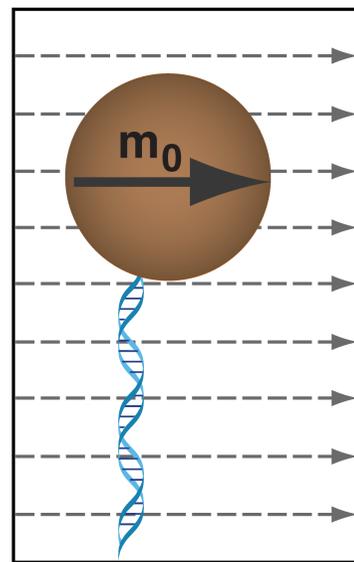
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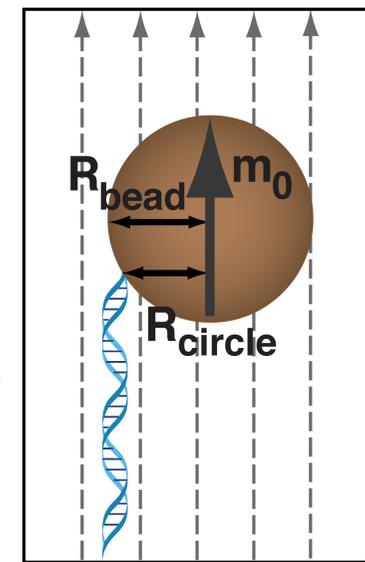
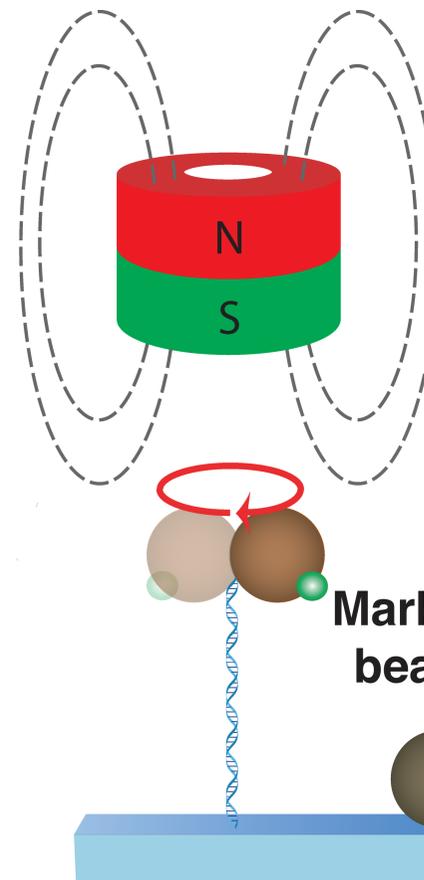
Torque tweezers:
 $\sigma_{\theta} \approx 5$ - 10°
 \Rightarrow Trap stiffness k_{θ} is 100-500 pN·nm/rad

Can we reduce the torsional trap of the magnets to zero?

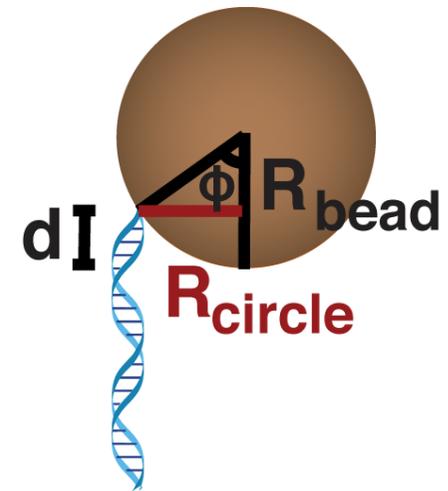
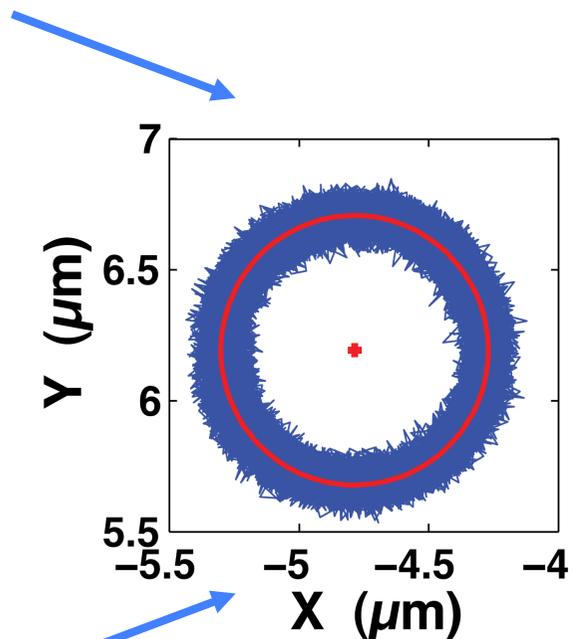
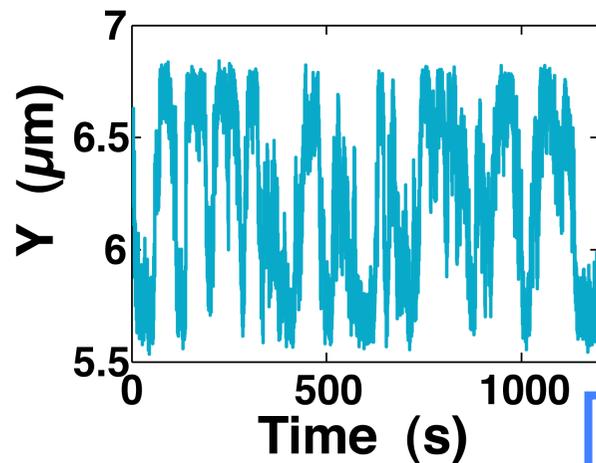
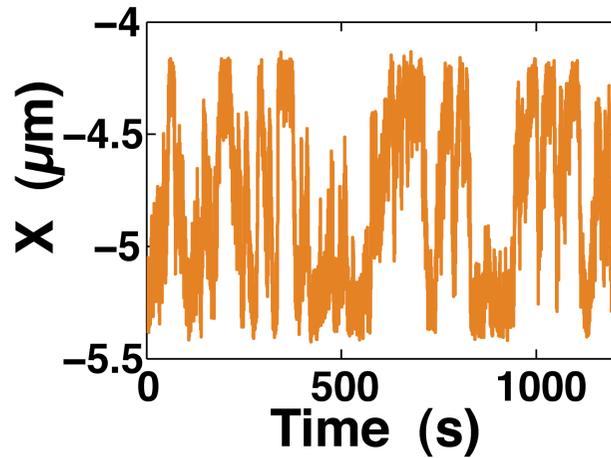
Conventional
Magnetic Tweezers



Freely Orbiting
Magnetic Tweezers



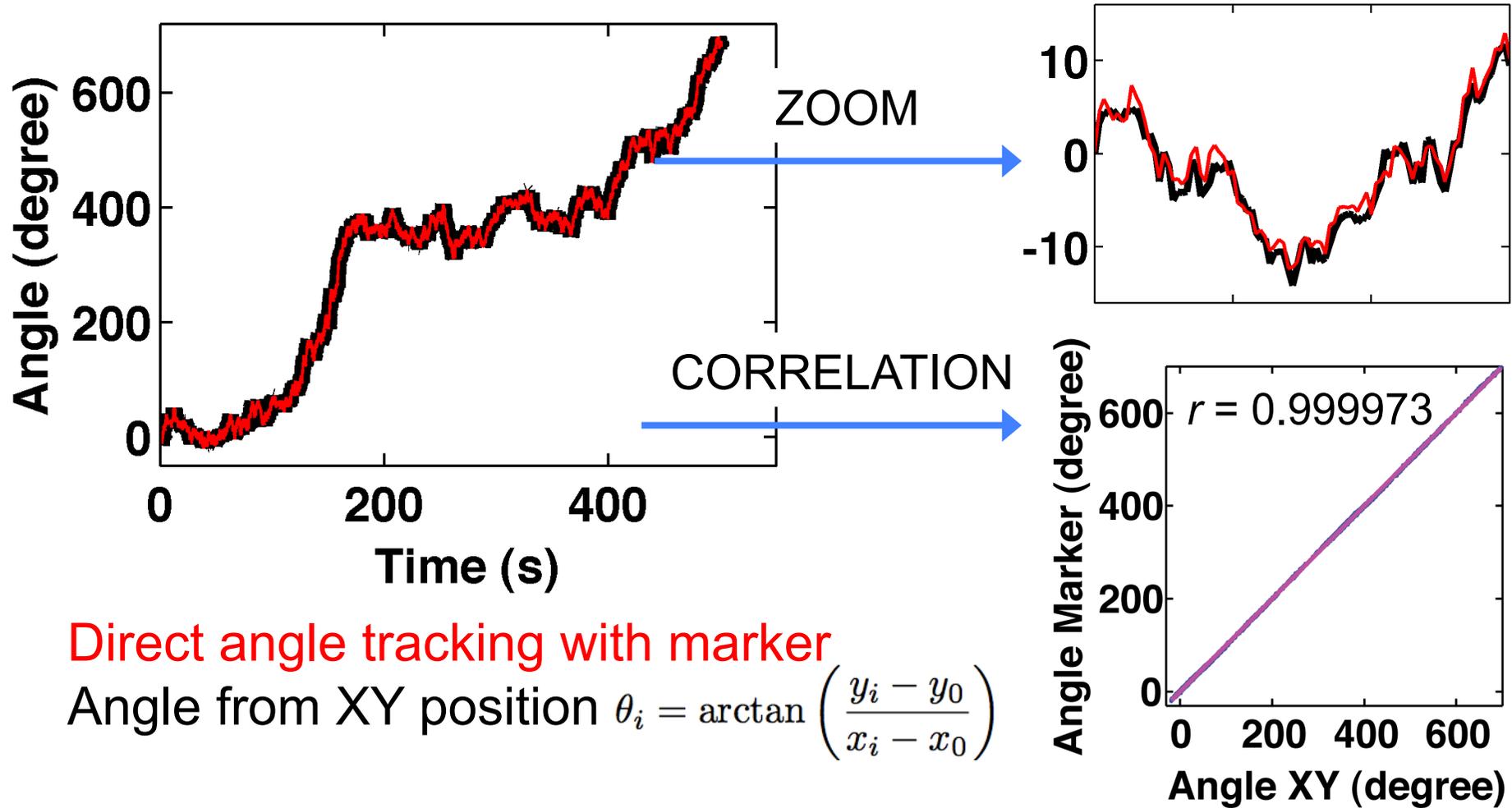
For a perfectly aligned cylindrical magnet, the fluctuations lie on a circle



Fit a circle with radius R_{circle} and center (x_0, y_0)

$$\min_{x_0, y_0, R} = \sum_i^N ((x_i - x_0)^2 + (y_i - y_0)^2 - R^2)^2$$

Tracking bead rotation from XY-position



Direct angle tracking with marker

Angle from XY position $\theta_i = \arctan\left(\frac{y_i - y_0}{x_i - x_0}\right)$

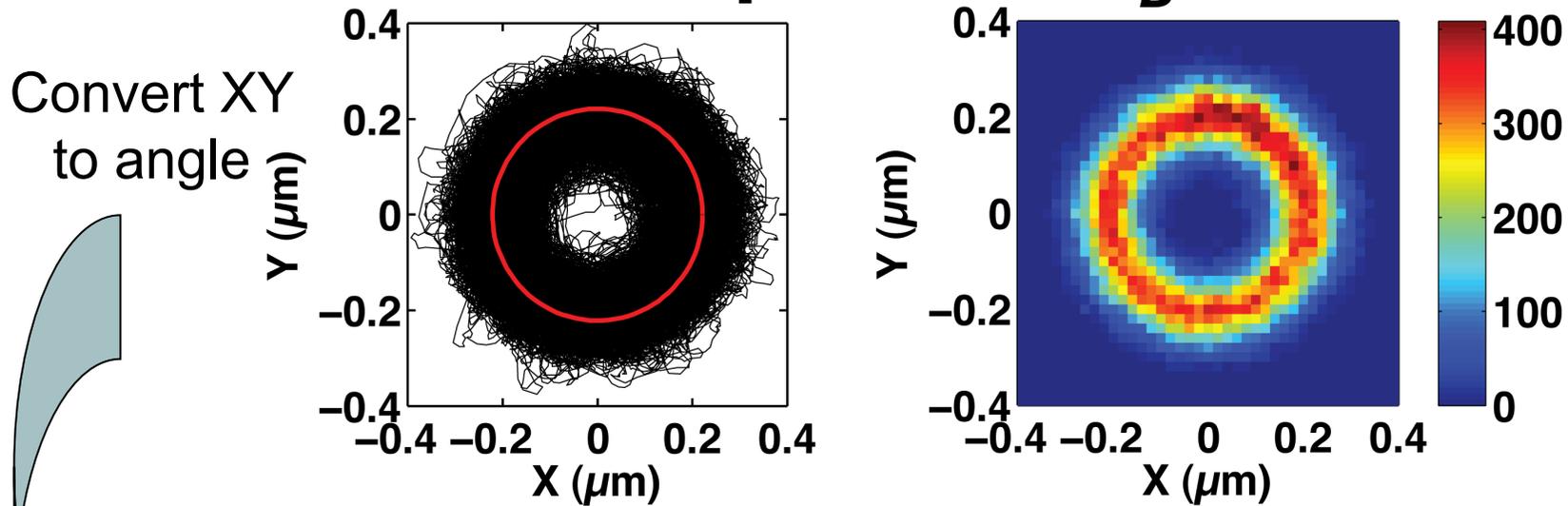
“Tracking angle without tracking angle”

“The art of fighting without fighting”

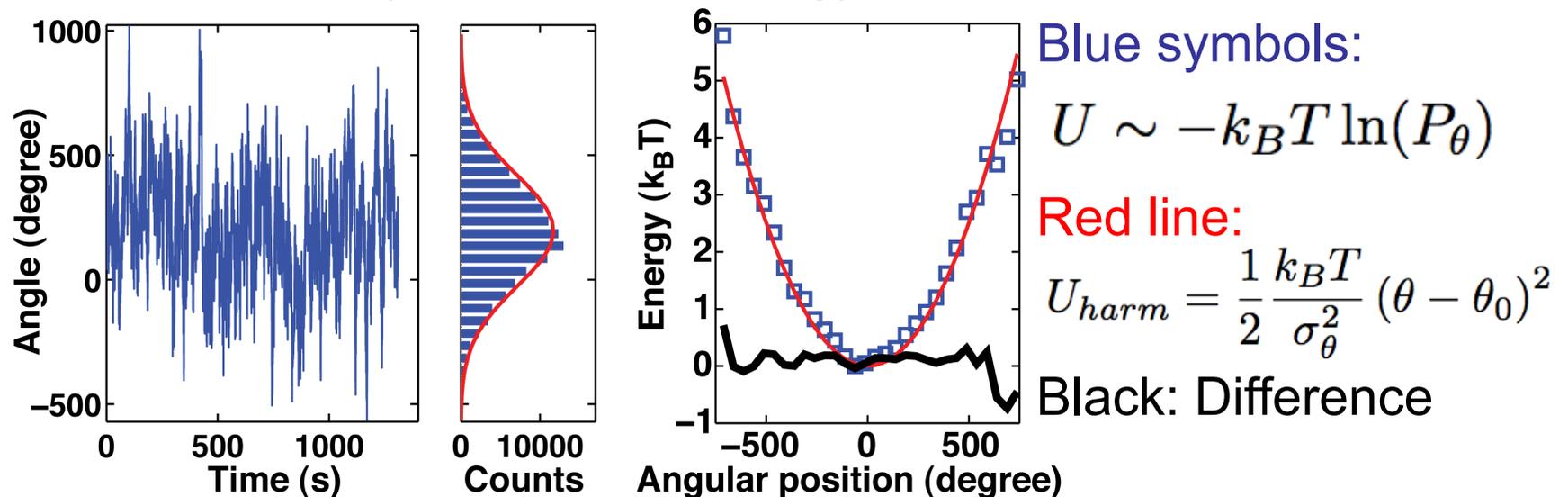


Bruce Lee, *Enter the Dragon* (1973)

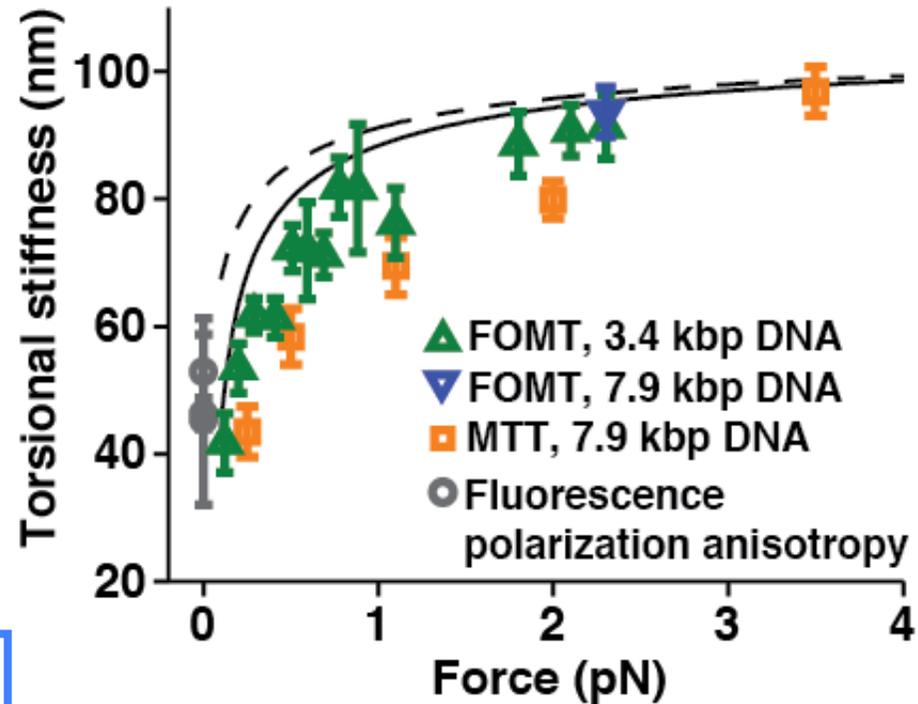
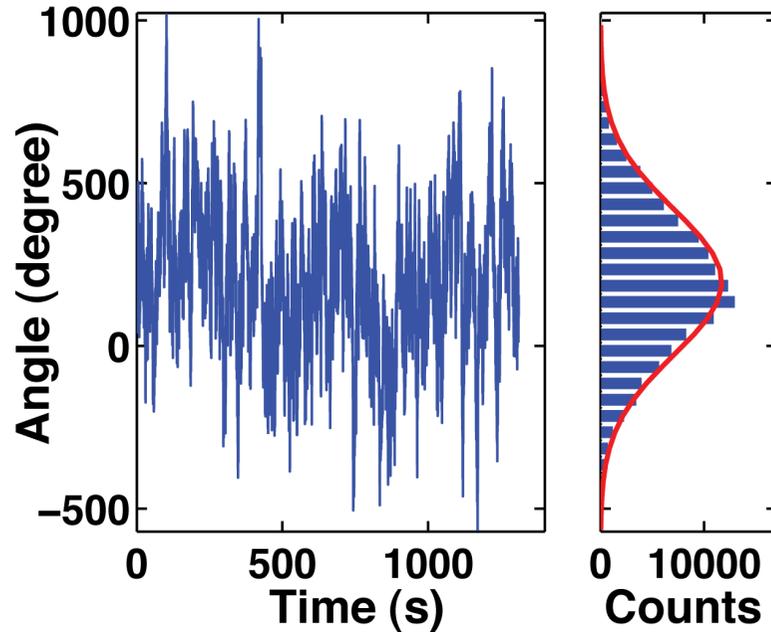
Torsional trap of the magnets is small compared to $k_B T$



Fluctuations imply a harmonic energy landscape



DNA torsional stiffness determined from angular fluctuations



Torsional stiffness C_{DNA} :

$$C_{DNA} = \frac{L_c}{\sigma_\theta^2}$$

Contour length $L_c = 1.15 \mu\text{m}$
 Fluctuations $\sigma_\theta = 223^\circ = 3.9 \text{ rad}$
 \Rightarrow Measured $C_{DNA} = 76 \text{ nm}$

Overall, we find consistent $C(F)$ values from FOMT and MTT measurements.

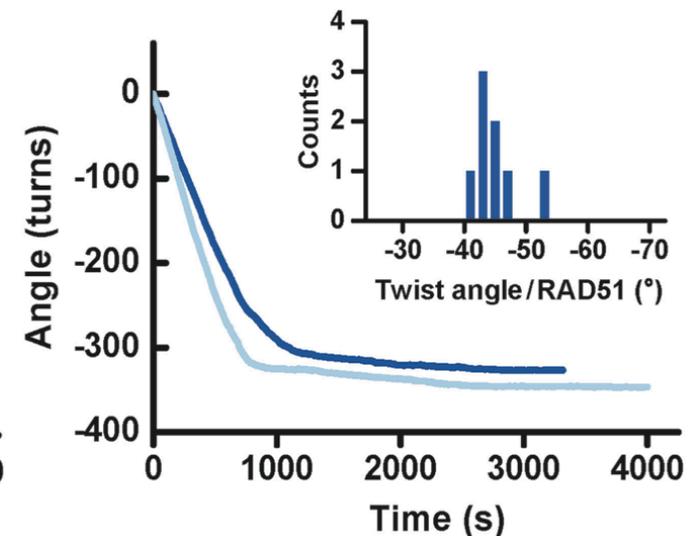
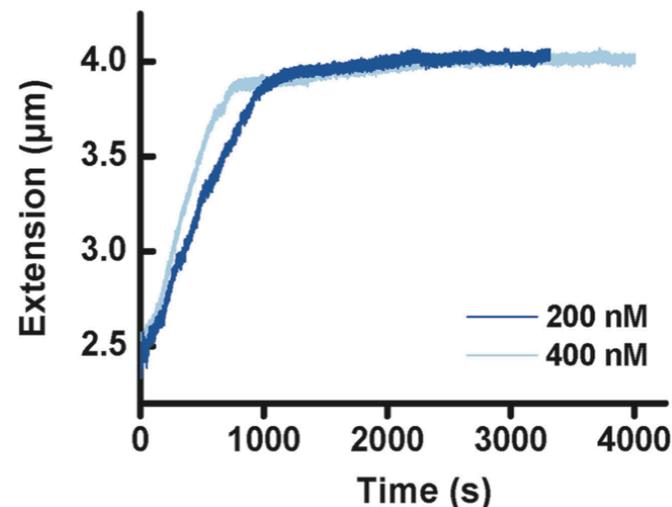
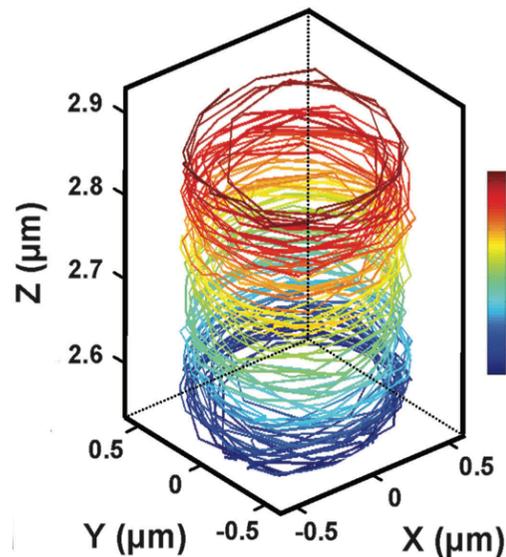
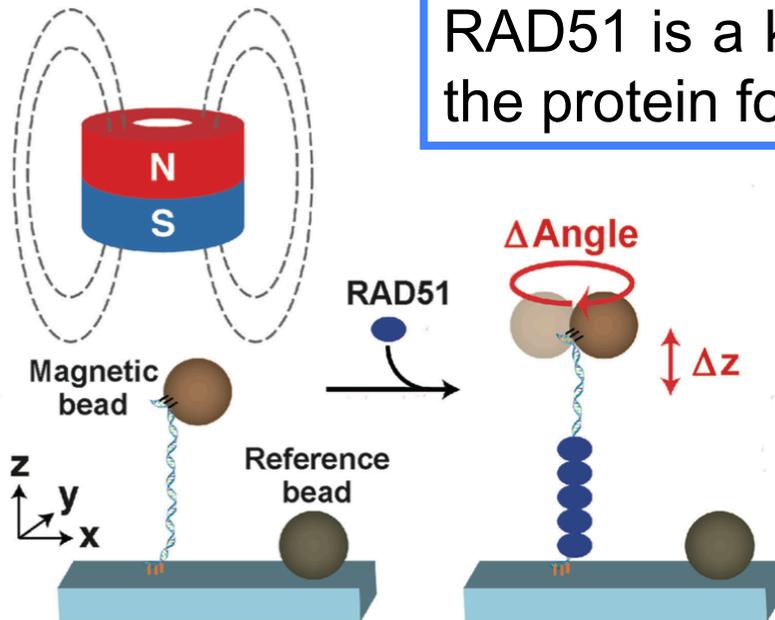
Lipfert[†], Wiggint[†], Kerssemakers, Pedaci & Dekker, *Nature Communications* (2011)

Studying the assembly and mechanical properties of RAD51-dsDNA filaments

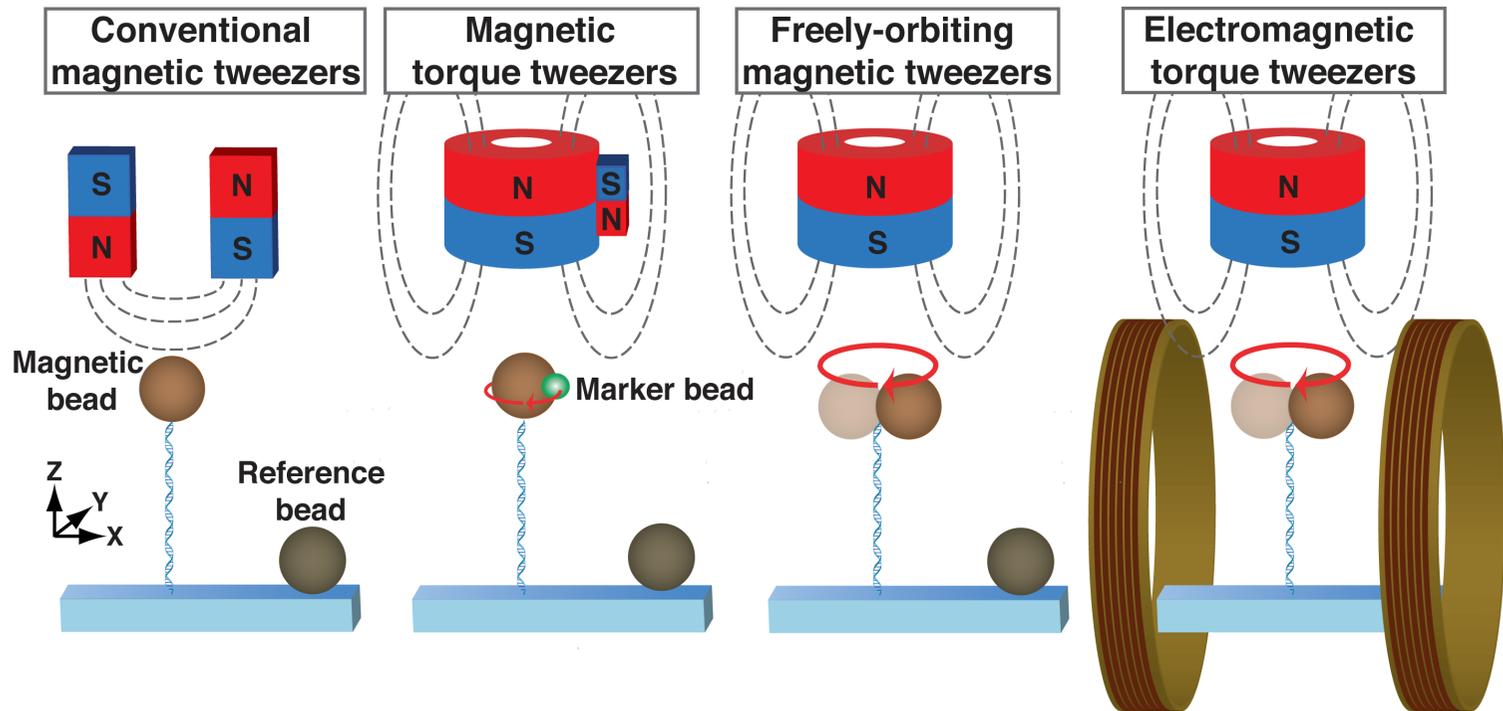
RAD51 is a key factor in homologous recombination; the protein forms filaments on ss and ds DNA

Monitor filament assembly in real time both in extension and angle; deduce the change in angle per monomer binding event.

Lee, Lipfert, Sanchez, Wyman & Dekker *NAR* (2013)



Novel Magnetic Tweezers for Direct Torque and Twist Measurements



Torque	apply only	apply & measure	free rotation	apply & measure or free
Angle tracking		✓	✓	✓
Force (apply & measure)	✓	✓	✓	✓
(x,y,z)-tracking	✓	✓	✓	✓

Strick, et al., Science (1996) Lipfert, et al., Nature Methods (2010) Lipfert[†], Wiggins[†], et al. Nature Commun. (2011) Janssen[†], Lipfert[†], et al. Nano Letters (2012)

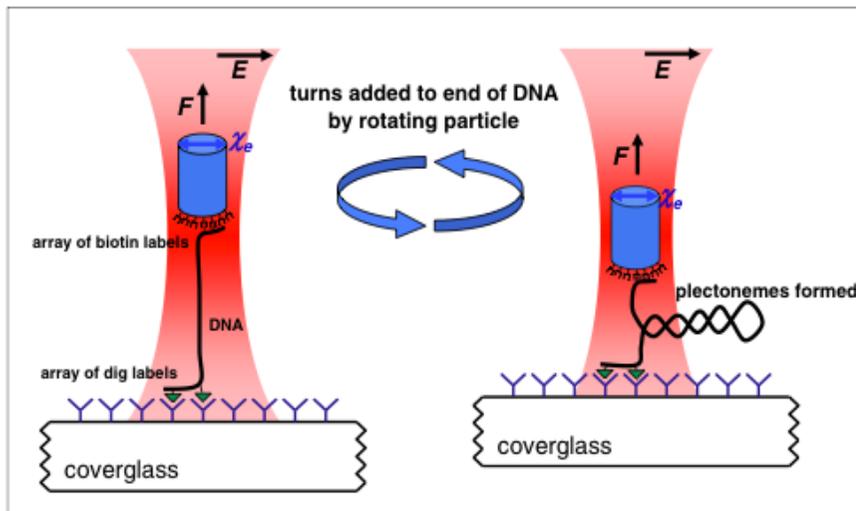
([†]Equal contribution) Lipfert, et al., Rev. Sci. Instrum. (2011) Patent (2010), Licensed (2012)

One sentence summary:

**Magnetic tweezers can apply
(and measure)
both **forces** and **torques**
to individual macromolecules**

Back up slides

Angular Trapping Strategies: Optical vs. Magnetic Torque Tweezers



La Porta & Wang *PRL* (2004)
Deufel et al. *Nature Methods* (2007)
Forth, et al. *PRL* (2008)

Similar approach:

Oroszi, Galajda, Kirei, Bottka & Ormos *PRL* (2006)

Optical torque tweezers:

+ High-bandwidth (kHz)

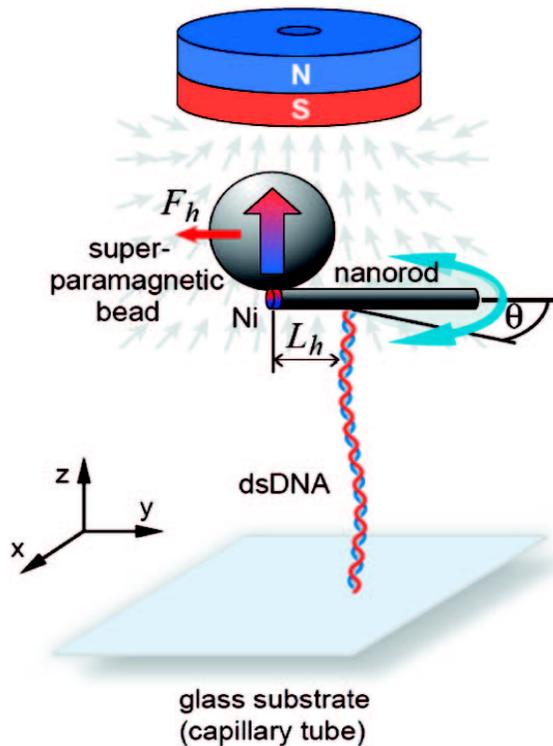
+ Better position resolution

– Hard to apply forces < 1 pN:
No negative supercoils!

– Relative complicated set up
and samples (nanofabrication)

– Difficult to multiplex

Angular Trapping Strategies: Nanorod Torque Tweezers

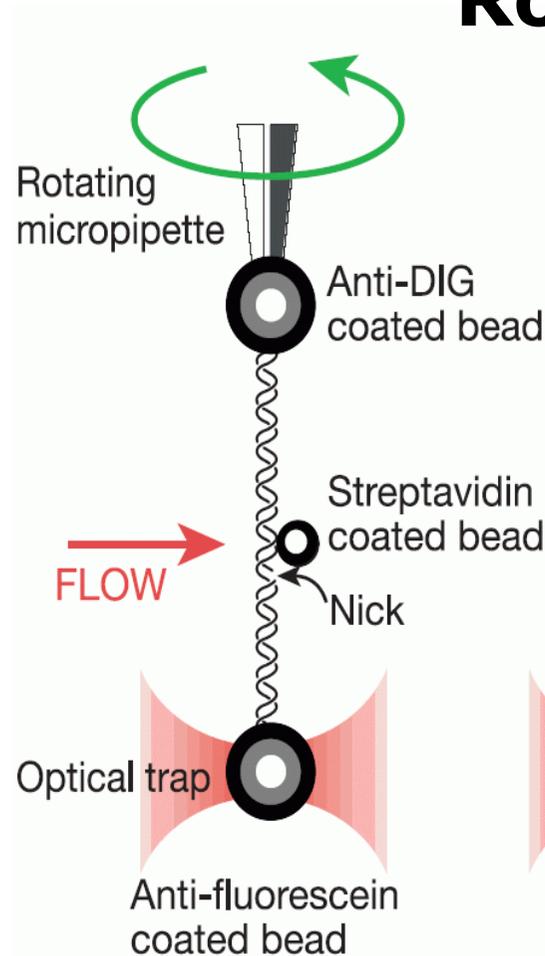


Overall similar to our scheme
Disadvantages of nanorod tweezers:

- Complicated sample assembly (nanofabricated rods + beads)
- Rotate by rotating the stage!
- Currently achieve only fairly poor resolution (~ 50 nm) and low forces (< 1.5 pN)

Celedon, et al. *Nanoletters* (2009)

Angular Trapping Strategies: Rotor Bead Tracking



Rotor bead tracking:

- + Can use small rotor beads, achieve good time resolution
- Difficult to multiplex
- Relies in specifically nicked DNA: Difficult for filaments, RNA, etc.

Bryant, *et al. Nature* (2003)

Using magnetic tweezers to rotate:

Gore, *et al. Nature* (2006), Oberstrass, *et al. PNAS* (2012)